Sparse-layer Inversion Using Basis Pursuit Decomposition

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Alternative Seismic Inversion Philosophies

• Sparse-spike methods invert the seismic trace for a limited number of reflection coefficients.
• This limits the thin-layer resolution.
• Alternatively, we formulate the seismic inversion problem to solve for a limited number of layer impulse pairs, rather than a limited number of spikes.
• Will this improve layer resolution?
Any reflection pair can be represented as the sum of even and odd impulse pairs:

\[
\text{= a} + b
\]

Thus: Any seismogram can be represented as the sum of even and odd responses.
Even and odd wedge dictionary elements
Basic Pursuit Dipole Decomposition

\[ r(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} (a_{n,m} \ast r_e(t, m, n, \Delta t) + b_{n,m} \ast r_o(t, m, n, \Delta t)) \]
Basic Pursuit Thin-Layer Response Decomposition

\[ s(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} (a_{n,m} \ast s_e(t, m, n, \Delta t) + b_{n,m} \ast s_o(t, m, n, \Delta t)) \]
Sparse Spike Inversion Results
Sparse-Layer Inversion Results
Effect Of Noise

(a) λ-correlation curves without noise

(b) λ-correlation curves with 40% noise
Maximum Correlations for 100 Reflectivity Series
Predominantly even wedge inversion without noise

Time thickness (ms)

(a) BPI inverted reflectivity with seismic wiggles overlaid
(b) SSI inverted reflectivity with seismic wiggles overlaid

(c) BPI Reflectivity Residual
(d) SSI Reflectivity Residual

- : Tuning thickness
\( r_1' \) : Inverted reflection coefficients at top of wedge
\( r_2' \) : Inverted reflection coefficients at base of wedge
Reflectivity residual : True reflection coefficients – inverted reflection coefficients

\( \lambda_{SPI} = 10^{-2} \)
\( \lambda_{SSI} = 10^{-7} \)
Predominantly odd wedge inversion without noise

Time thickness (ms)

(a) BPI inverted reflectivity with seismic wiggles overlaid
(b) SSI inverted reflectivity with seismic wiggles overlaid

(c) Reflectivity Residual
(d) Reflectivity Residual

- Tuning thickness
- $r_1'$: Inverted reflection coefficients at top of wedge
- $r_2'$: Inverted reflection coefficients at base of wedge
- Reflectivity residual: True reflection coefficients – inverted reflection coefficients
Predominantly even wedge inversion with 10% noise

Time thickness (ms)

(a) BPI inverted reflectivity with seismic wiggles overlaid

(b) SSI inverted reflectivity with seismic wiggles overlaid

(c) Reflectivity residual

(d) Reflectivity residual

\[ \lambda_{\text{BPI}} = 2 \]

\[ \lambda_{\text{SSI}} = 10^{-3} \]

- **Tuning thickness**
- **\( r_1' \)**: Inverted reflection coefficients at top of wedge
- **\( r_2' \)**: Inverted reflection coefficients at base of wedge

Reflectivity residual: True reflection coefficients – inverted reflection coefficients
Real Data Example
2.1 BPI inverted relative impedance with seismic wiggles overlaid

2.2 SSI inverted relative impedance with seismic wiggles overlaid
BPI inverted relative impedance with seismic wiggles overlaid

SSI inverted relative impedance with seismic wiggles overlaid
Conclusions

• Sparse-layer inversion can be accomplished by basis pursuit of a dictionary of functions representing thin-bed reflectivity patterns.

• This method determines a sparse number of thin-layer responses that sum to form the seismic trace.

• Synthetic tests indicate that sparse-layer inversion using basis pursuit (BPI) can better resolve thin beds than a comparable sparse-spike inversion (SSI) and usually correlates better to known reflectivity when optimal regularization parameters are used for both methods.