Sparse-layer Inversion Using Basis Pursuit Decomposition

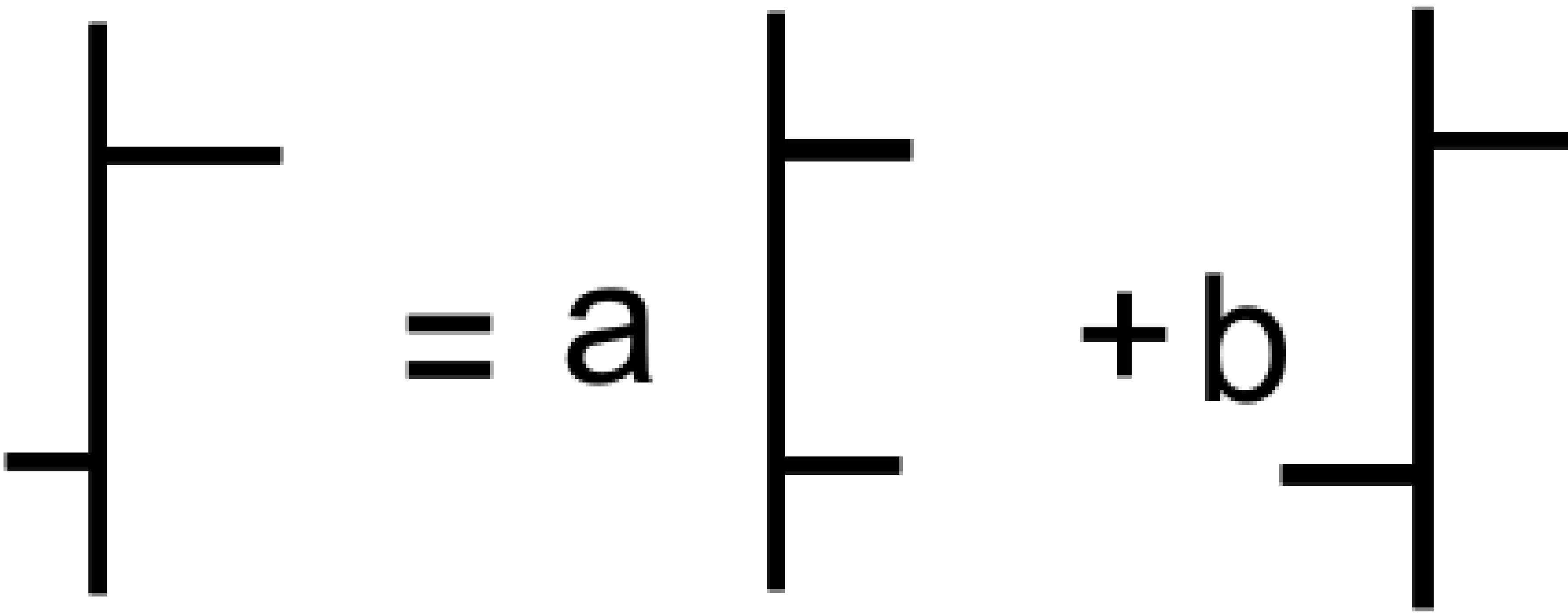
John Castagna and Rui Zhang

University of Houston

Alternative Seismic Inversion Philosophies

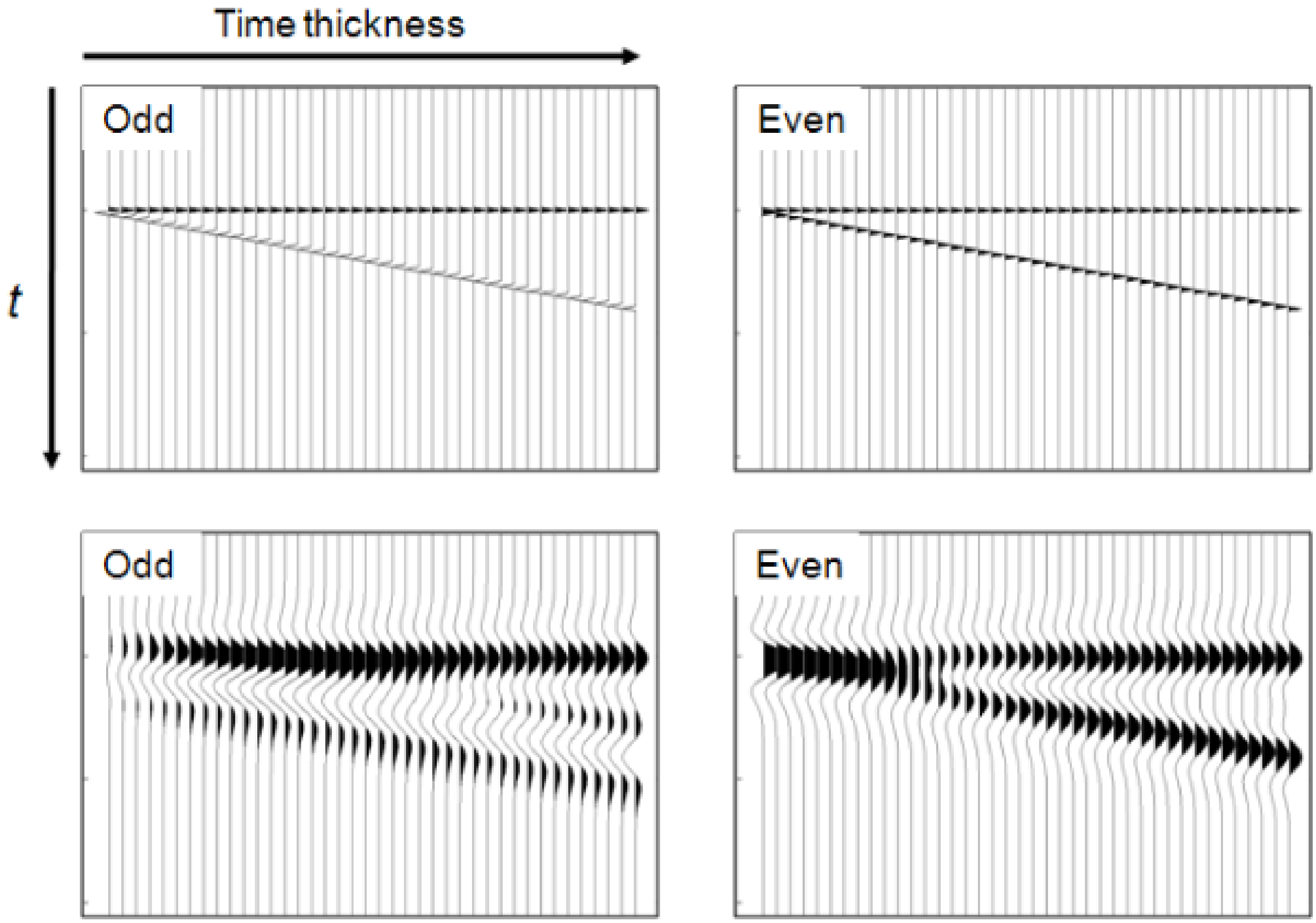
- Sparse-spike methods invert the seismic trace for a limited number of reflection coefficients.
- This limits the thin-layer resolution.
- Alternatively, we formulate the seismic inversion problem to solve for a limited number of layer impulse pairs, rather than a limited number of spikes.
- Will this improve layer resolution?

Any reflection pair can be represented as the sum of even and odd impulse pairs:



Thus: Any seismogram can be represented as the sum of even and odd responses.

Even and odd wedge dictionary elements

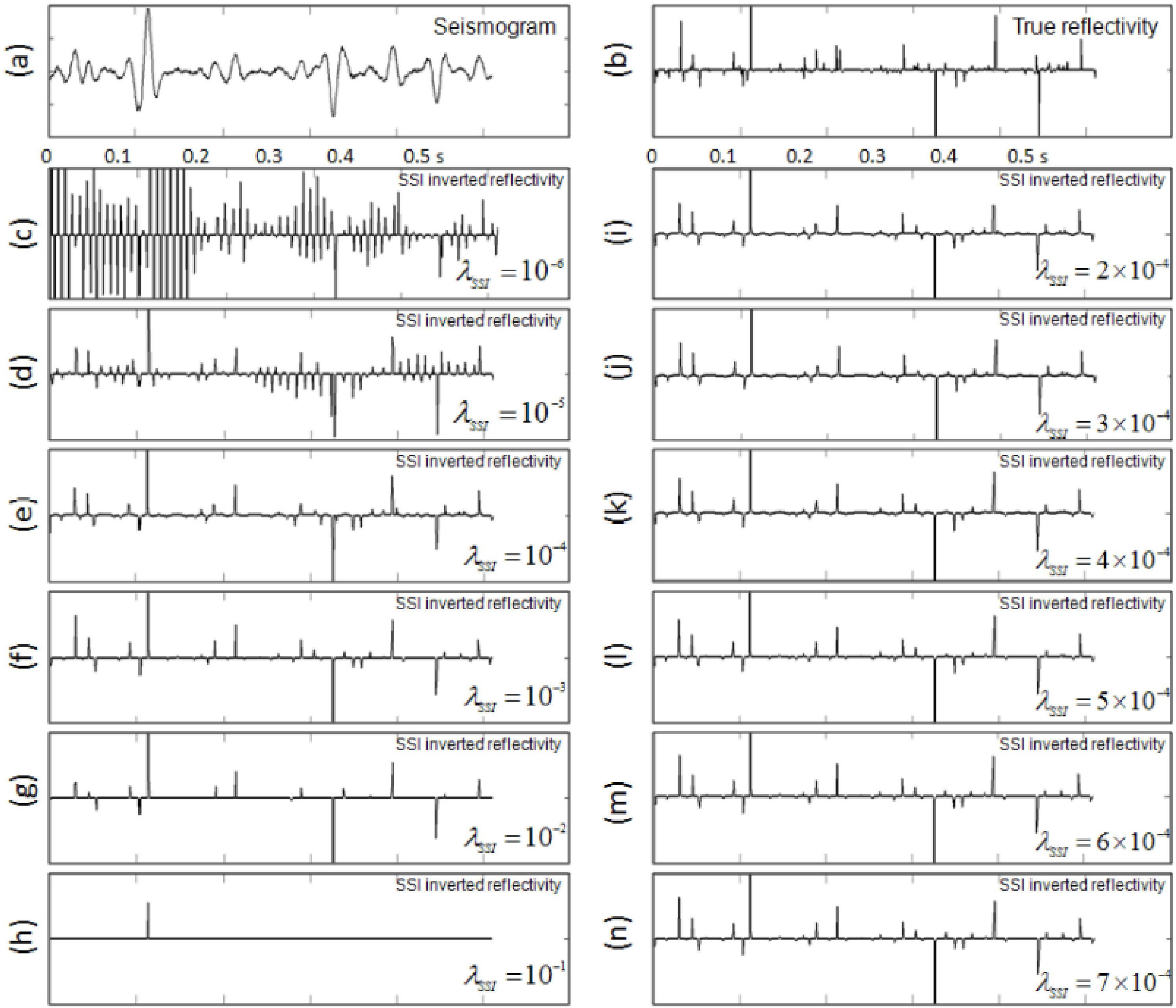


Basic Pursuit Dipole Decompostion

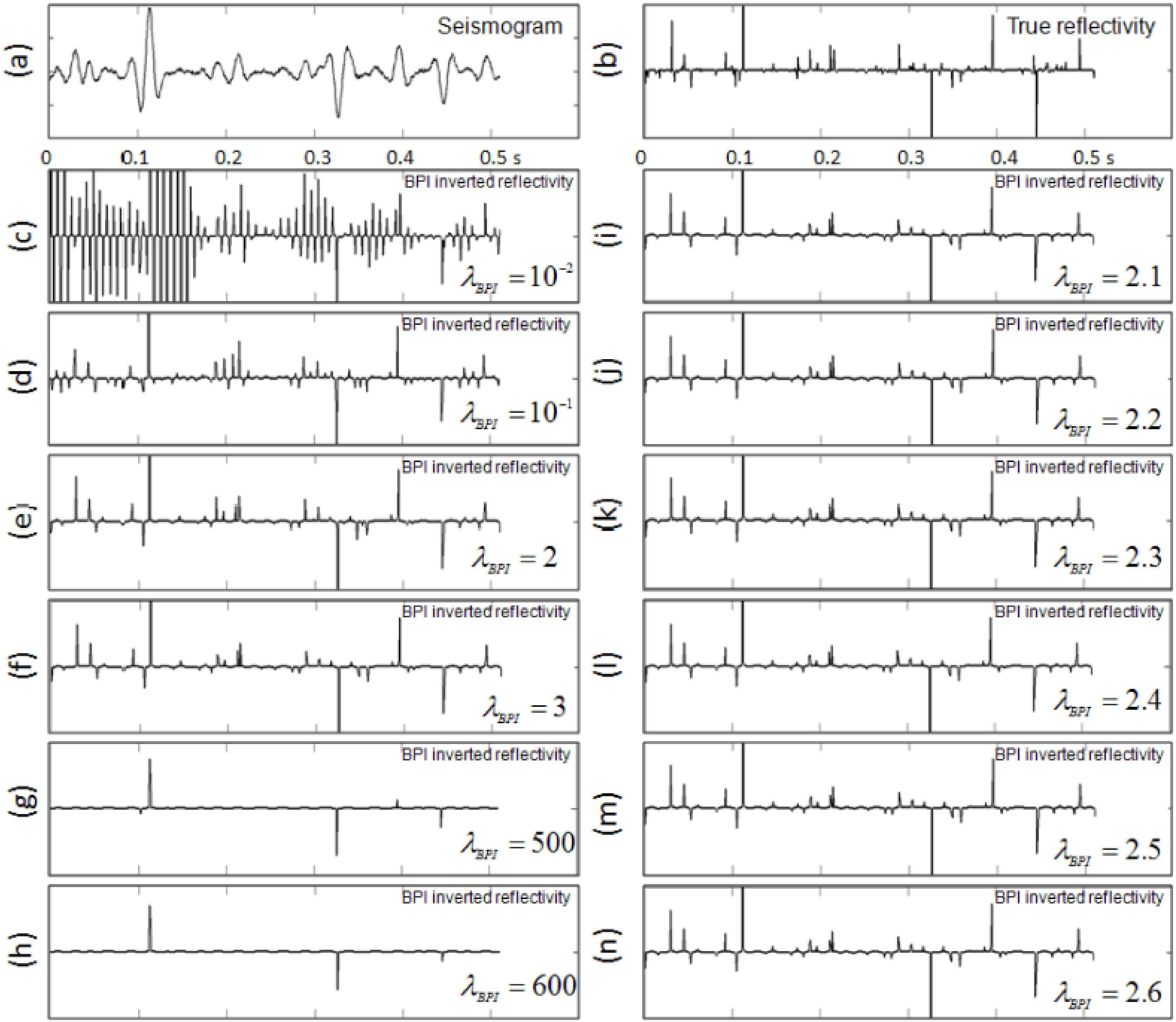
$$r(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} (a_{n,m} * r_e(t, m, n, \Delta t) + b_{n,m} * r_o(t, m, n, \Delta t))$$

Basic Pursuit Thin-Layer Response Decompostion

$$s(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} (a_{n,m} * s_e(t, m, n, \Delta t) + b_{n,m} * s_o(t, m, n, \Delta t))$$

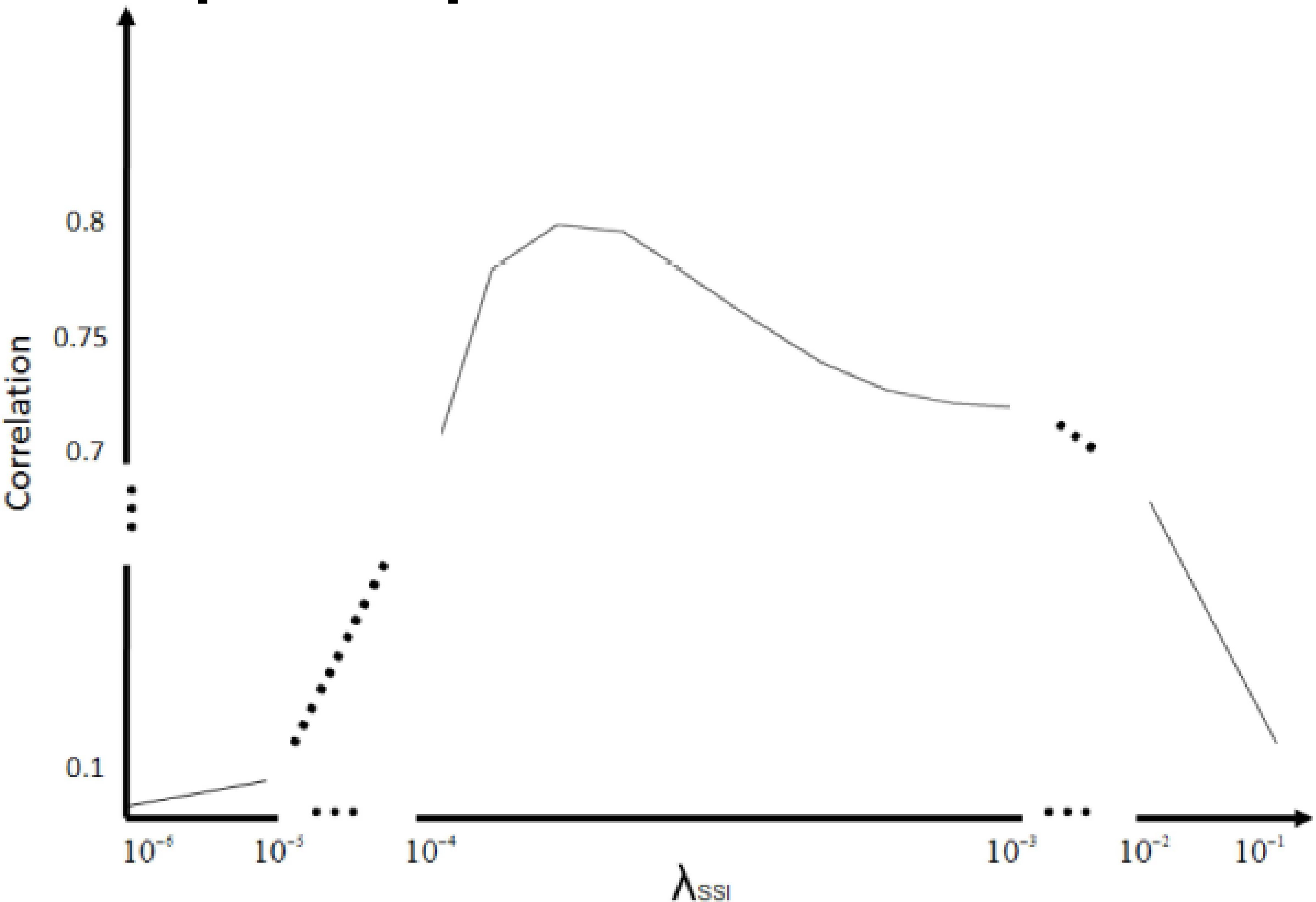


(a)
Seismogram
(b)
True reflectivity
0
0.1
0.2
0.3
0.4
0.5 s
(c)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-6}$
(d)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-5}$
(e)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-4}$
(f)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-3}$
(g)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-2}$
(h)
SSI inverted reflectivity
$\lambda_{SSI} = 10^{-1}$
(i)
SSI inverted reflectivity
$\lambda_{SSI} = 2\times10^{-4}$
(j)
SSI inverted reflectivity
$\lambda_{SSI} = 3\times10^{-4}$
(k)
SSI inverted reflectivity
$\lambda_{SSI} = 4\times10^{-4}$
(l)
SSI inverted reflectivity
$\lambda_{SSI} = 5\times10^{-4}$
(m)
SSI inverted reflectivity
$\lambda_{SSI} = 6\times10^{-4}$
(n)
SSI inverted reflectivity
$\lambda_{SSI} = 7\times10^{-4}$

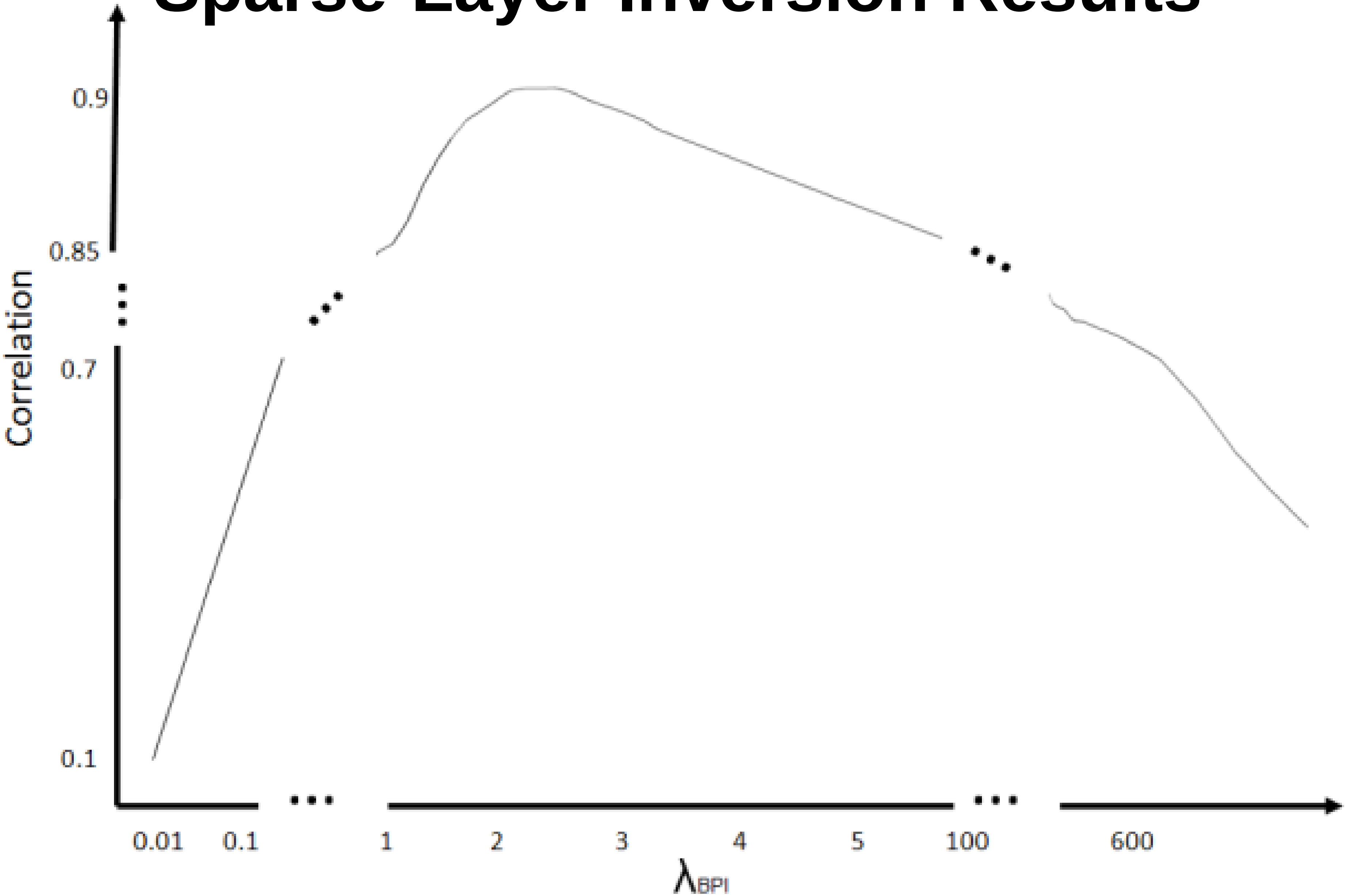


(a)
Seismogram
(b)
True reflectivity
0
0.1
0.2
0.3
0.4
0.5 s
0
0.1
0.2
0.3
0.4
0.5 s
(c)
BPI inverted reflectivity
$\lambda_{BPI} = 10^{-2}$
(d)
BPI inverted reflectivity
$\lambda_{BPI} = 10^{-1}$
(e)
BPI inverted reflectivity
$\lambda_{BPI} = 2$
(f)
BPI inverted reflectivity
$\lambda_{BPI} = 3$
(g)
BPI inverted reflectivity
$\lambda_{BPI} = 500$
(h)
BPI inverted reflectivity
$\lambda_{BPI} = 600$
(i)
BPI inverted reflectivity
$\lambda_{BPI} = 2.1$
(j)
BPI inverted reflectivity
$\lambda_{BPI} = 2.2$
(k)
BPI inverted reflectivity
$\lambda_{BPI} = 2.3$
(l)
BPI inverted reflectivity
$\lambda_{BPI} = 2.4$
(m)
BPI inverted reflectivity
$\lambda_{BPI} = 2.5$
(n)
BPI inverted reflectivity
$\lambda_{BPI} = 2.6$

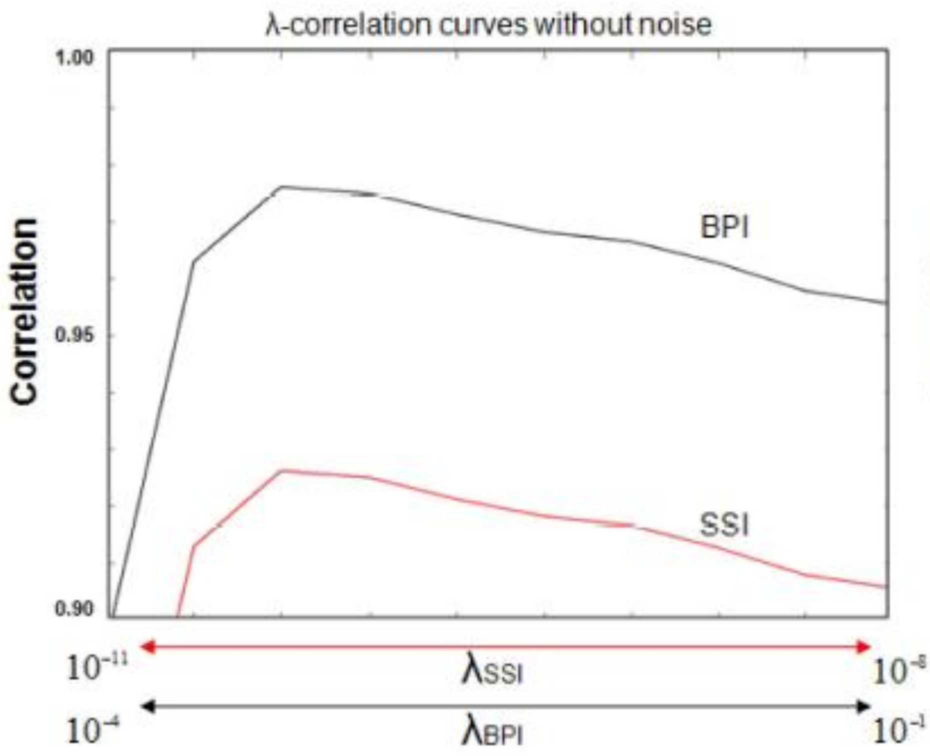
Sparse Spike Inversion Results



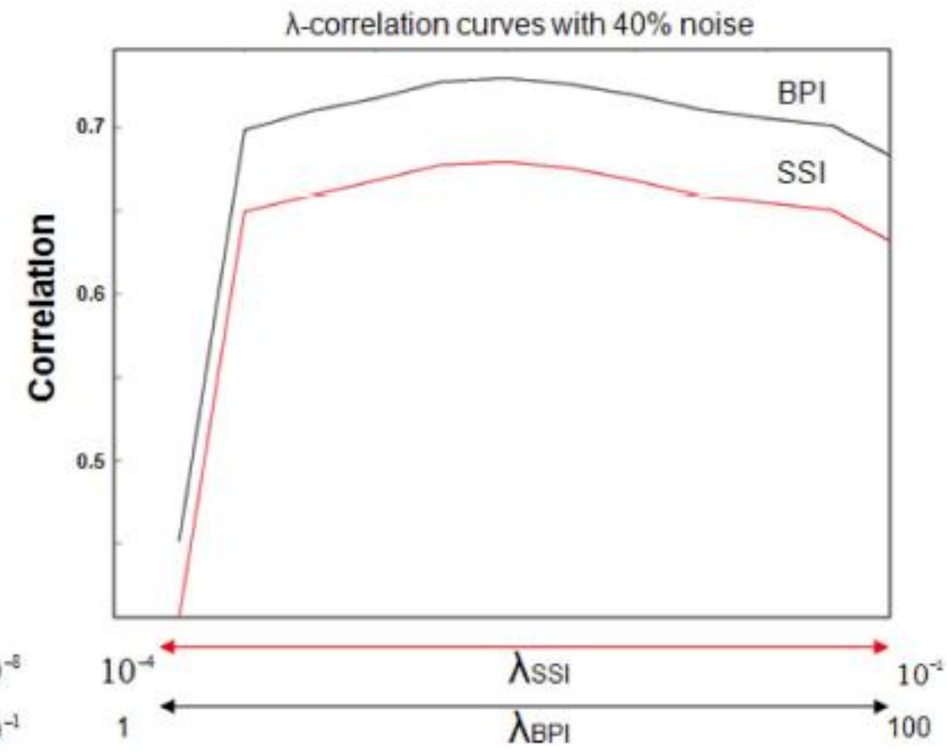
Sparse-Layer Inversion Results



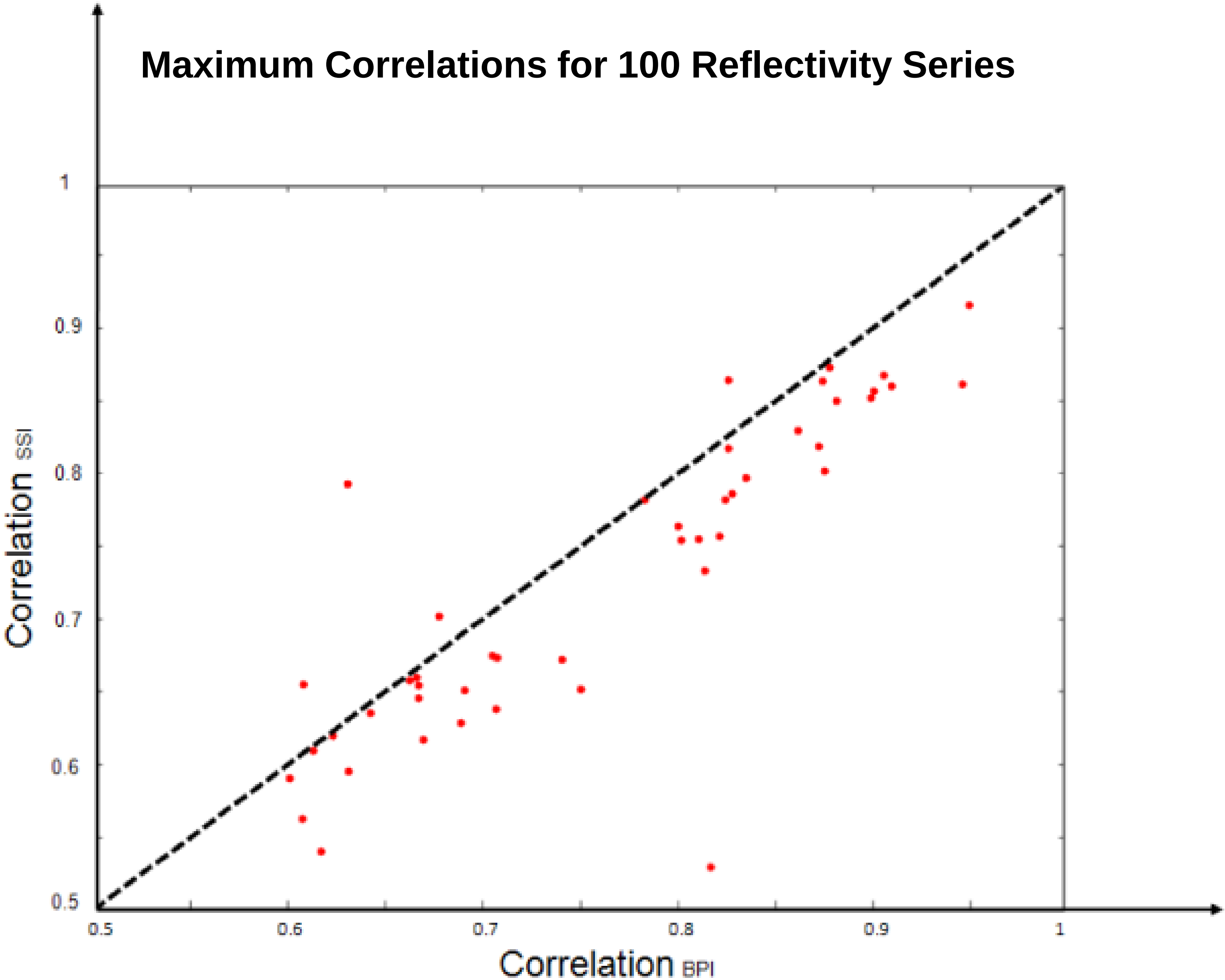
Effect Of Noise



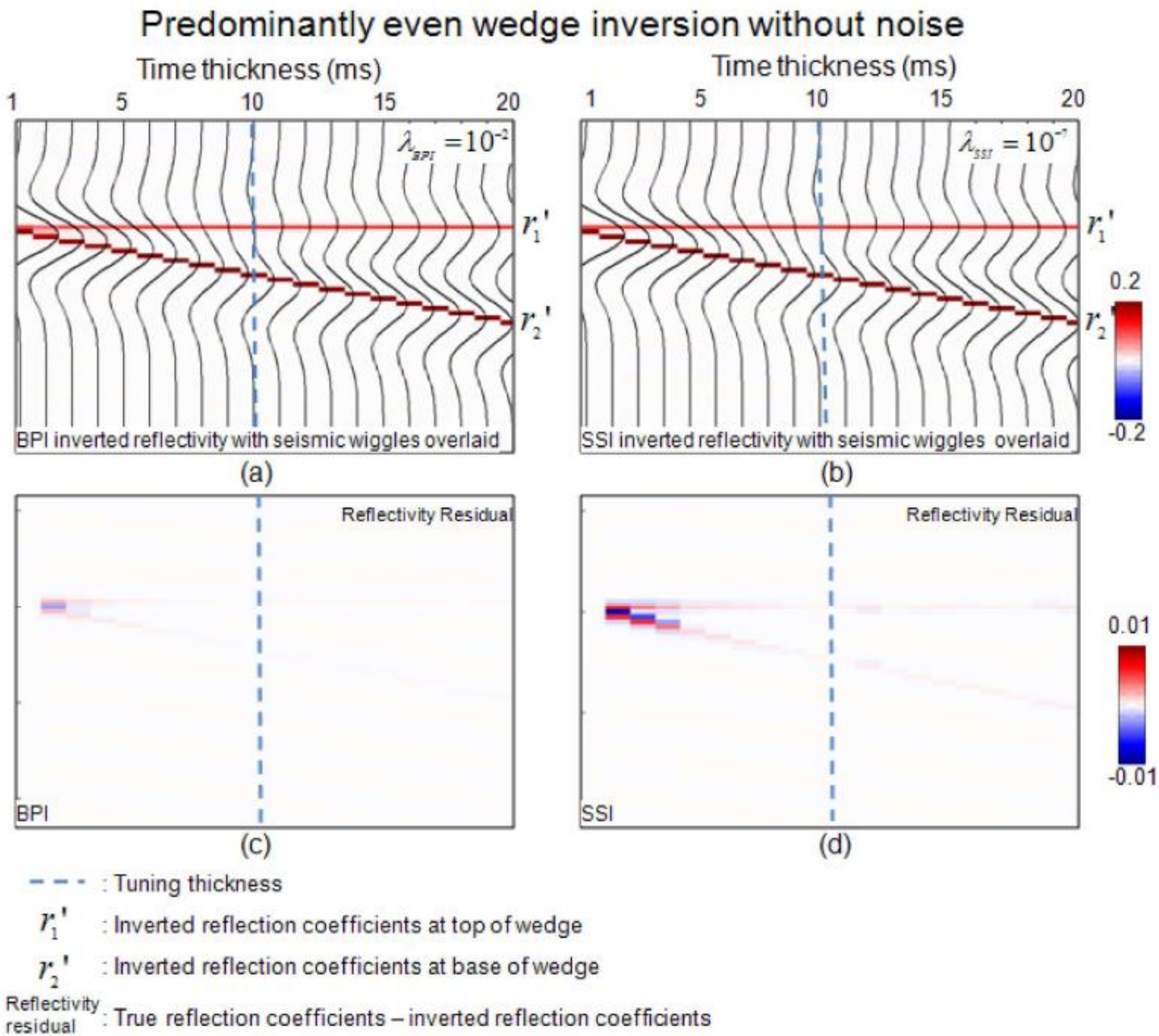
(a)



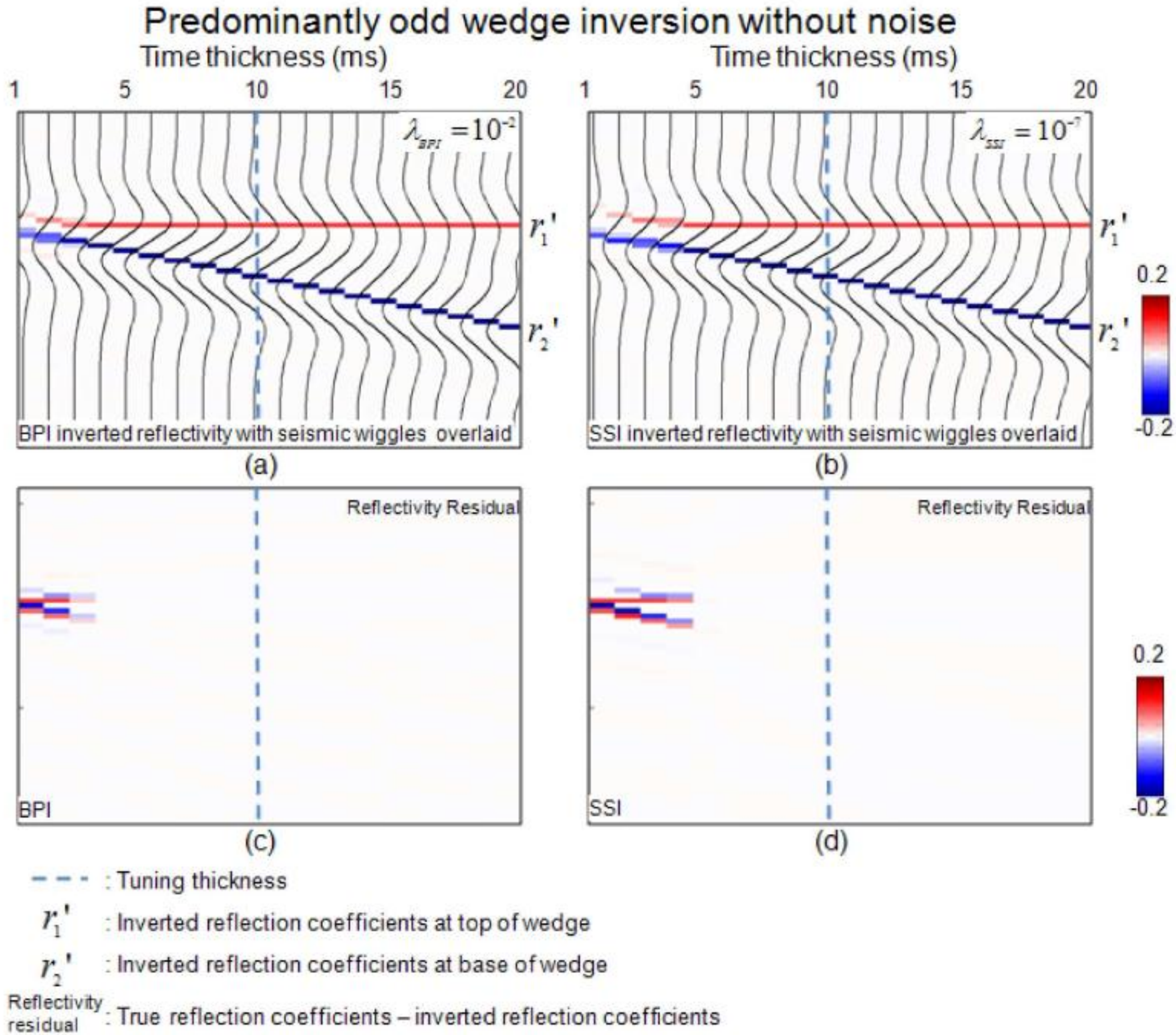
(b)



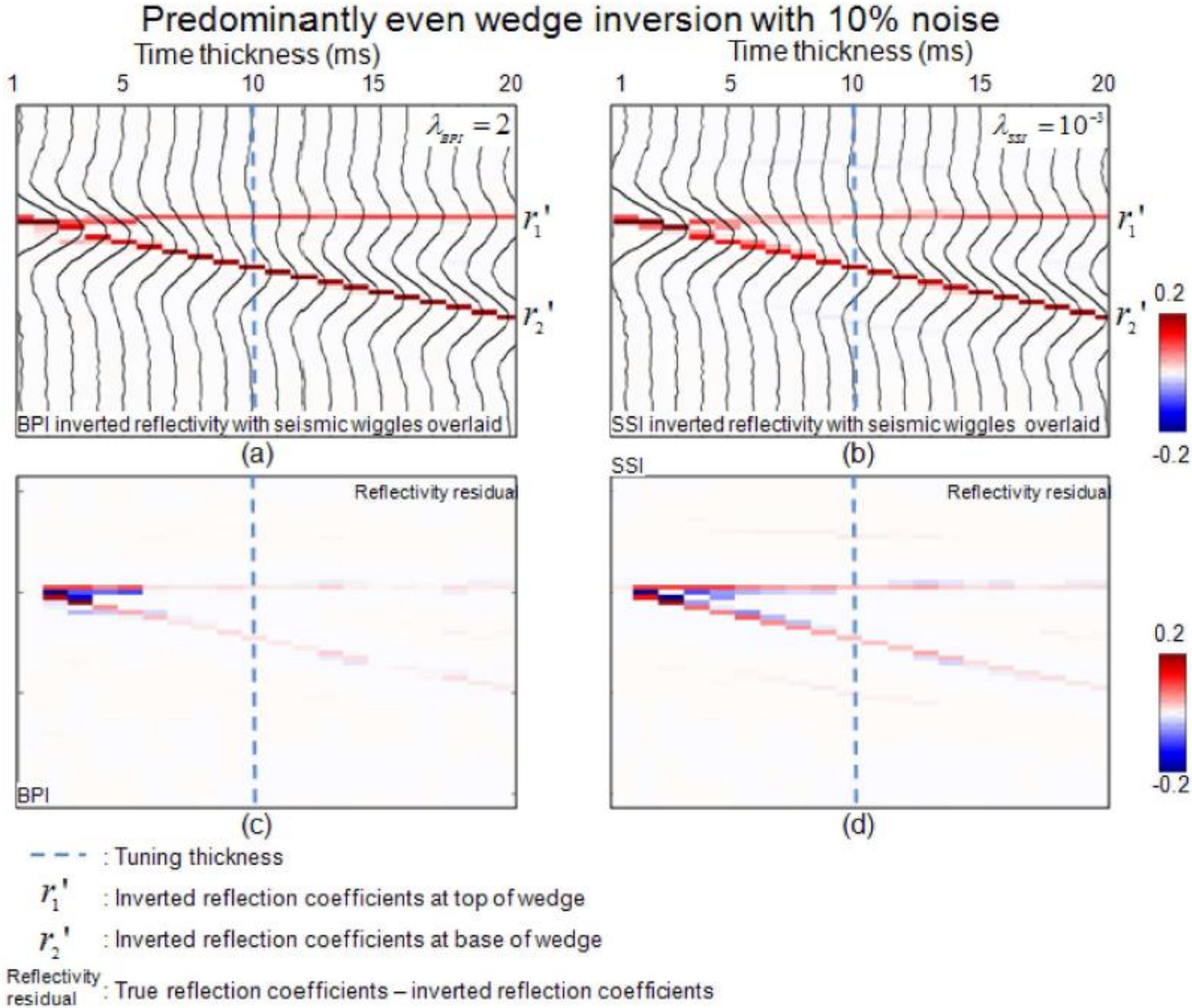
Maximum Correlations for 100 Reflectivity Series
Correlation SSI
Correlation BPI
1
0.9
0.8
0.7
0.6
0.5
0.5
0.6
0.7
0.8
0.9
1

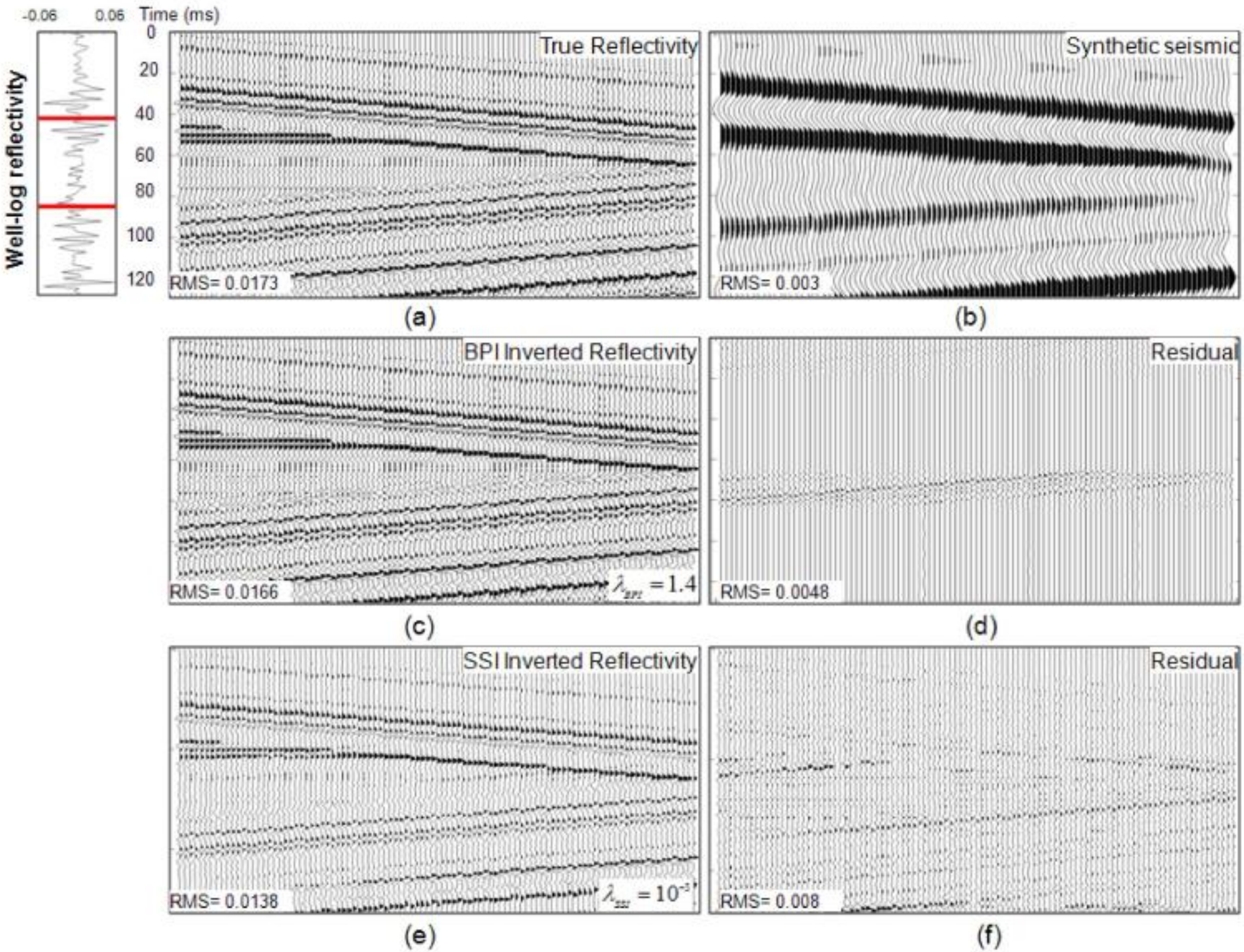


Predominantly even wedge inversion without noise
Time thickness (ms)
1
5
10
15
20
$\lambda_{BPI} = 10^{-2}$
r_1'
r_2'
BPI inverted reflectivity with seismic wiggles overlaid
(a)
Time thickness (ms)
1
5
10
15
20
$\lambda_{SSI} = 10^{-7}$
r_1'
r_2'
SSI inverted reflectivity with seismic wiggles overlaid
(b)
0.2
-0.2
Reflectivity Residual
BPI
(c)
Reflectivity Residual
SSI
(d)
0.01
-0.01
: Tuning thickness
r_1' : Inverted reflection coefficients at top of wedge
r_2' : Inverted reflection coefficients at base of wedge
Reflectivity residual : True reflection coefficients – inverted reflection coefficients



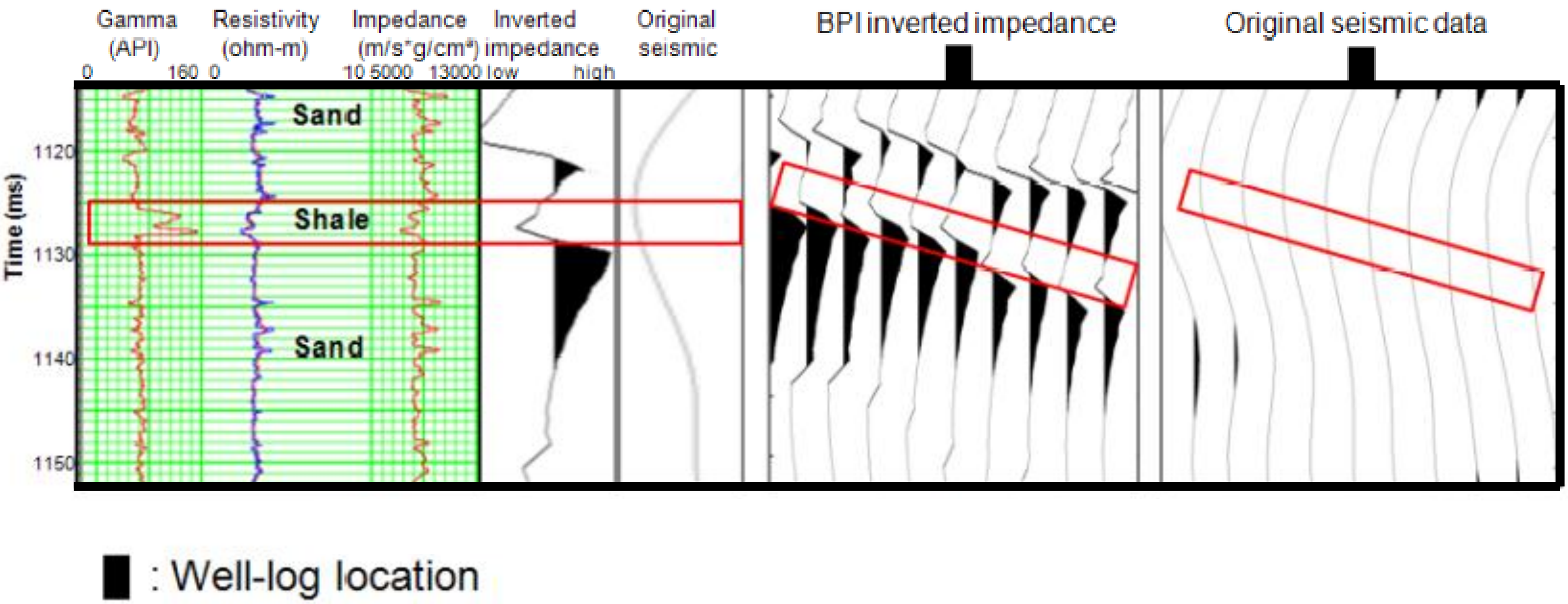
Predominantly odd wedge inversion without noise
Time thickness (ms)
1
5
10
15
20
$\lambda_{BPI} = 10^{-2}$
r_1'
r_2'
BPI inverted reflectivity with seismic wiggles overlaid
(a)
Time thickness (ms)
1
5
10
15
20
$\lambda_{SSI} = 10^{-7}$
r_1'
r_2'
SSI inverted reflectivity with seismic wiggles overlaid
(b)
0.2
-0.2
Reflectivity Residual
BPI
(c)
Reflectivity Residual
SSI
(d)
0.2
-0.2
: Tuning thickness
r_1' : Inverted reflection coefficients at top of wedge
r_2' : Inverted reflection coefficients at base of wedge
Reflectivity residual : True reflection coefficients – inverted reflection coefficients

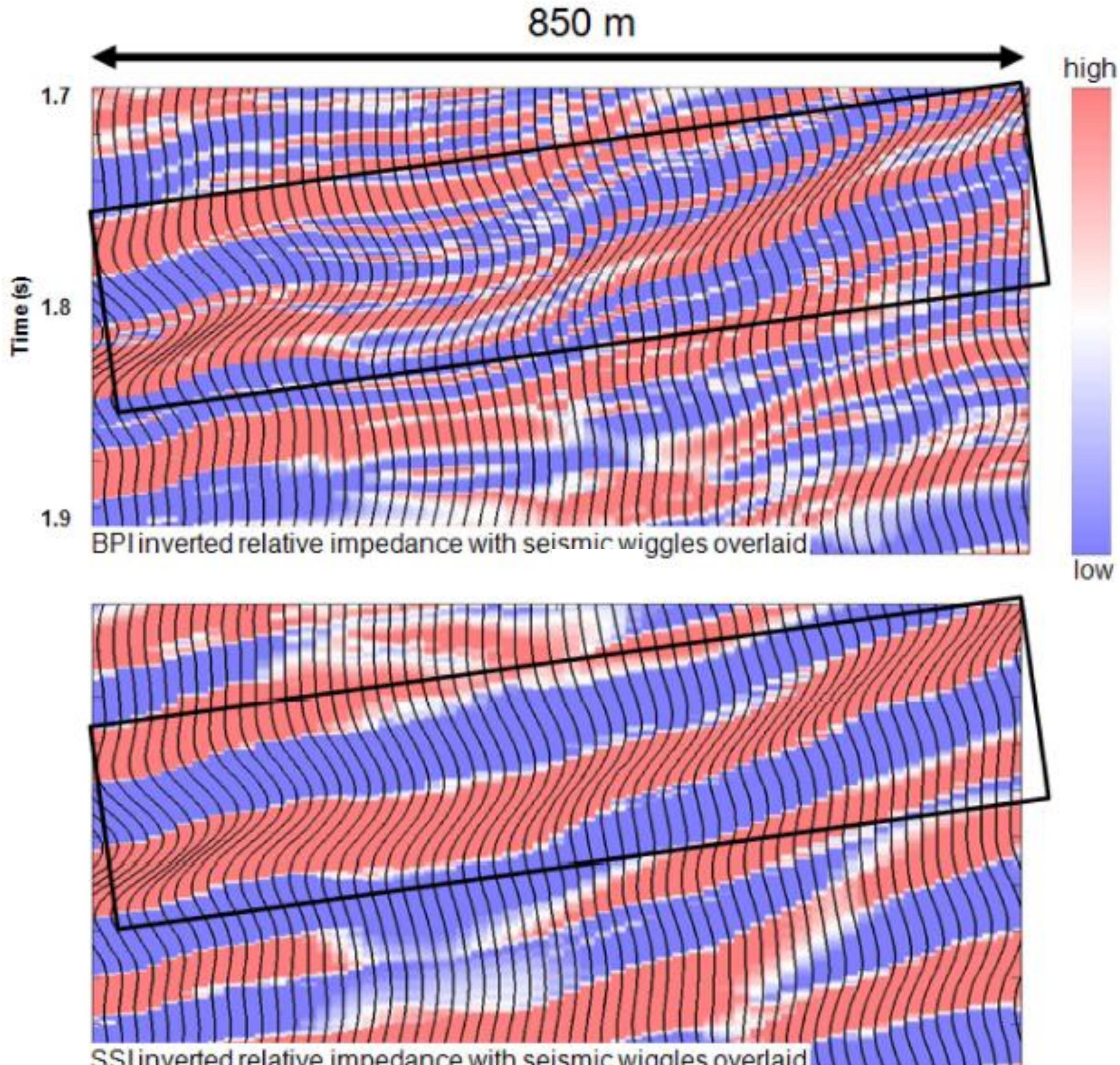

Predominantly even wedge inversion with 10% noise
Time thickness (ms)
1
5
10
15
20
$\lambda_{BPI} = 2$
r_1'
r_2'
BPI inverted reflectivity with seismic wiggles overlaid
(a)
Time thickness (ms)
1
5
10
15
20
$\lambda_{SSI} = 10^{-3}$
r_1'
r_2'
SSI inverted reflectivity with seismic wiggles overlaid
(b)
0.2
-0.2
Reflectivity residual
BPI
(c)
SSI
Reflectivity residual
(d)
0.2
-0.2
: Tuning thickness
r_1' : Inverted reflection coefficients at top of wedge
r_2' : Inverted reflection coefficients at base of wedge
Reflectivity residual : True reflection coefficients – inverted reflection coefficients



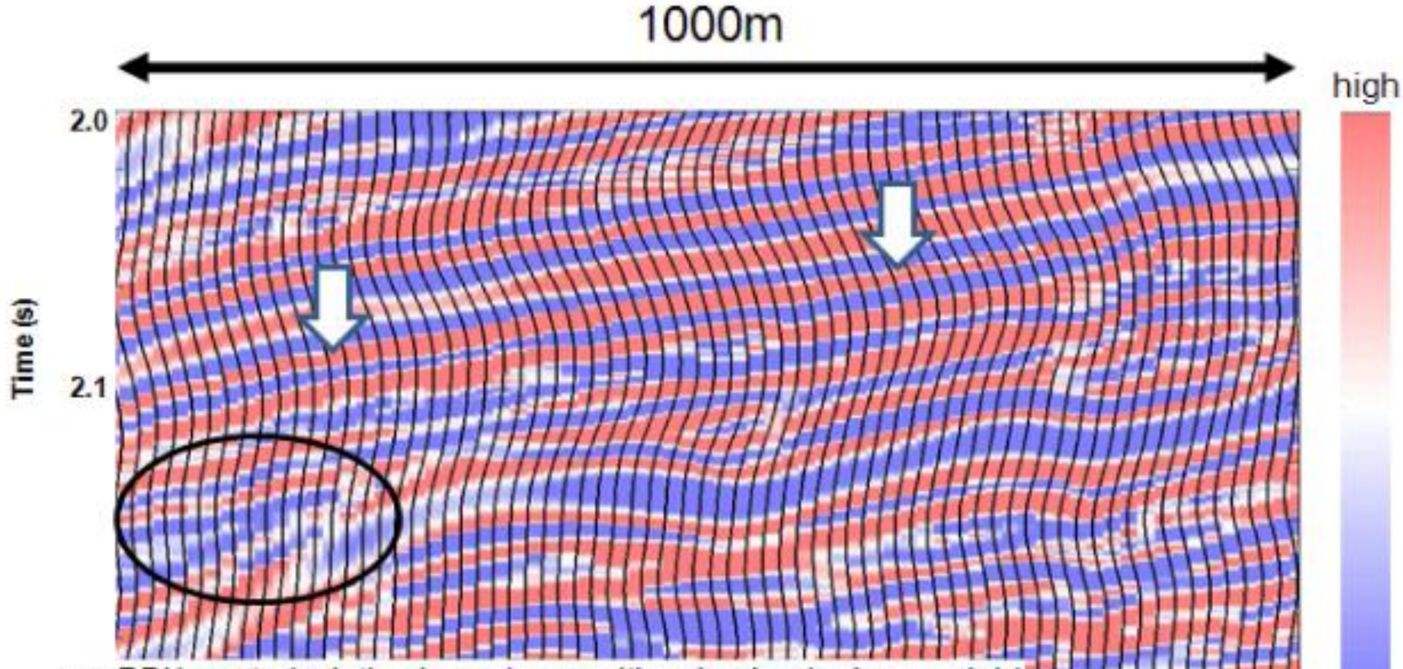
Well-log reflectivity
-0.06
0.06
Time (ms)
0
20
40
60
80
100
120
True Reflectivity
RMS= 0.0173
Synthetic seismic
RMS= 0.003
(a)
(b)
BPI Inverted Reflectivity
RMS= 0.0166
$\lambda_{BPI} = 1.4$
Residual
RMS= 0.0048
(c)
(d)
SSI Inverted Reflectivity
RMS= 0.0138
$\lambda_{SSI} = 10^{-5}$
Residual
RMS= 0.008
(e)
(f)

Real Data Example

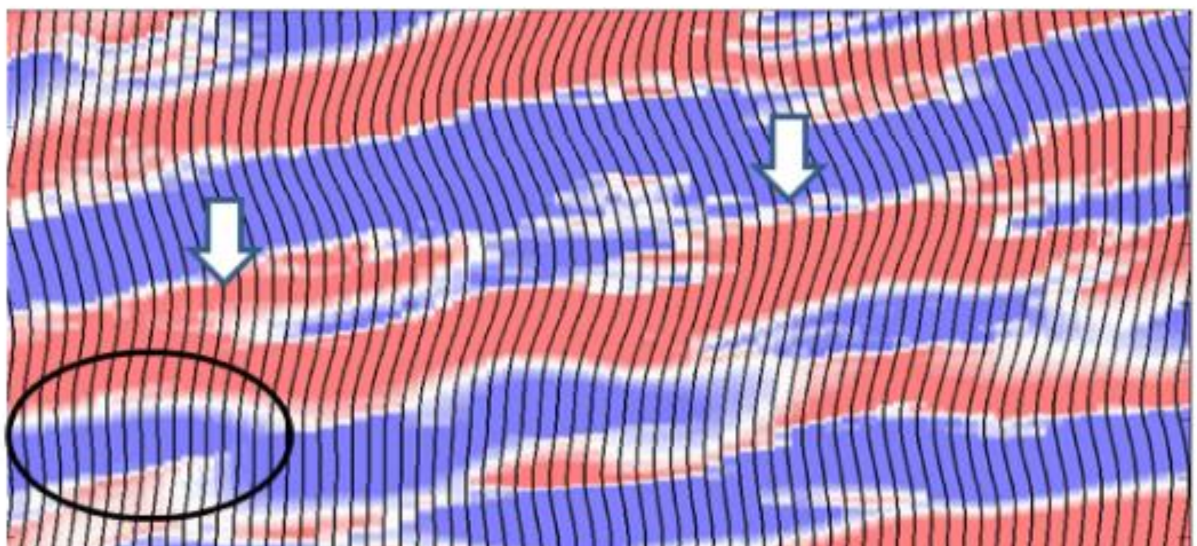




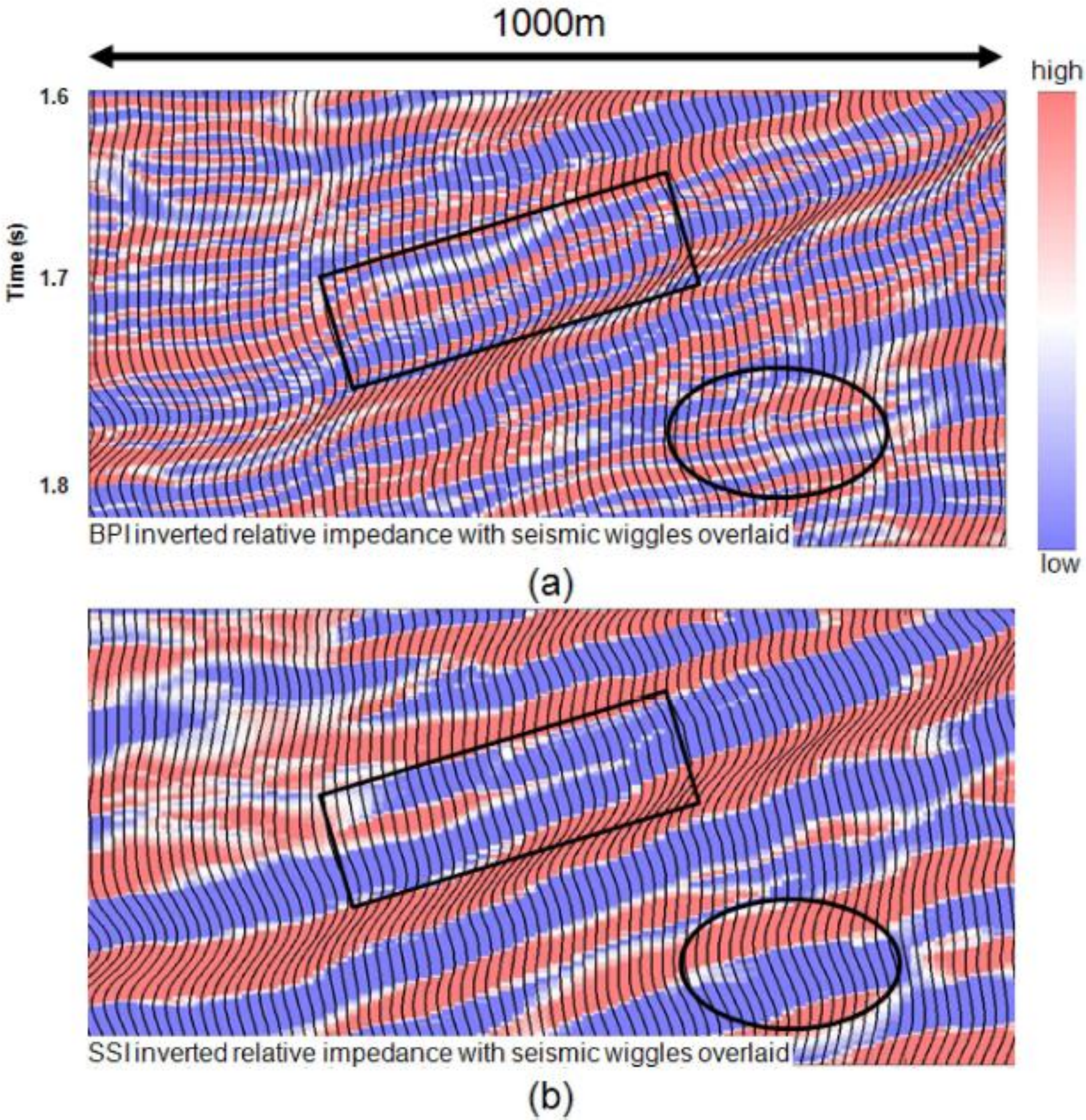
850 m
1.7
1.8
1.9
Time (s)
high
low
BPI inverted relative impedance with seismic wiggles overlaid
SSI inverted relative impedance with seismic wiggles overlaid



BPI inverted relative impedance with seismic wiggles overlaid



SSI inverted relative impedance with seismic wiggles overlaid


1000m
high
low
1.6
1.7
1.8
Time (s)
BPI inverted relative impedance with seismic wiggles overlaid
(a)
SSI inverted relative impedance with seismic wiggles overlaid
(b)

Conclusions

•Sparse-layer inversion can be accomplished by basis pursuit of a dictionary of functions representing thin-bed reflectivity patterns.

•This method determines a sparse number of thin-layer responses that sum to form the seismic trace.

•Synthetic tests indicate that sparse-layer inversion using basis pursuit (BPI) can better resolve thin beds than a comparable sparse-spike inversion (SSI) and usually correlates better to known reflectivity when optimal regularization parameters are used for both methods.