Energy reassignment of an image for improved dispersion curve picking

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Section I: Executive Summary

The energy re assignment imaging (ERI) technique proposed in this paper improves image processing and picking within a surface-wave noise mitigation suite GRB-3D (Ground Roll Buster 3D), developed by Sunwoong Lee and Warren Ross (Lee et al., 2008). GRB-3D is specifically designed to mitigate surface waves in spatially inhomogeneous media by aligning the surface waves, thereby separating them from seismic reflections, using phase matching techniques and dynamic phase-velocity estimates. This paper presents a brief examination of dispersive surface waves to provide background for both GRB-3D and ERI. The ERI technique is applied early in the GRB-3D operation flow to the image of dispersive surface-wave energy in the frequency-slowness domain. Subsequently, the algorithm used by GRB-3D to image the dispersive energy is briefly explained and a discussion is presented on the complications of imaging dispersive energy in heterogeneous areas. The basic theory behind ERI is presented and ERI is shown to improve the phase-velocity estimates from the frequency-slowness domain. To protect the interests of ExxonMobil and illustrate the operation of GRB-3D and ERI a derived dataset created from field data is used. Development for ERI was done at the ExxonMobil Upstream Research Company.

Section II: Statement of Problem and Initial Objectives

Estimating spatially varying dispersion curves is fundamental for determining parameters needed to mitigate surface waves in GRB-3D. To account for heterogeneity across large surveys a dynamic method is used to window seismic traces across the survey and image the dispersive energy unique to each window. GRB-3D is designed for large surveys which may require thousands of windowed areas; therefore, picking the dispersion curve from the image is automated with a picking algorithm. An averaging algorithm is used to combine the multiple dispersion curves from the overlapping windows to create a volume representing velocity and
frequency properties spatially. The resulting volume is used for a phase-matched filtering process in GRB-3D.

Phase velocity estimation by beamforming is used to form the power spectrum of dispersive energy at each particular window of the survey. Beamforming provides a robust method for calculating dispersion curves for surveys where large variation in velocity and frequency are expected. Smaller windowed traces are desirable as they capture spatial variation for a survey at a finer resolution. However, as explored later, the resolution of the beamformed field (frequency versus slowness velocity) is inversely proportional to window size, resulting in smearing of the imaged dispersion beam as window size decreases. Larger window sizes provide higher resolution for dispersion curve imaging; however, if the window encompasses an area which exhibits strong heterogeneity in shear wave velocity, as shown in Figure 1, more than one fundamental mode may exist for the imaged dispersive energy.

![Shear Wave Velocity at 10m depth](image1.png)

**Fig. 1.** Spatial shear wave velocity model inverted from field data at a 10 m depth. Black box encompasses windowed traces shown as points. (a) Beamformed dispersive modes from shear wave velocity model; white line denotes the automated picking. (b) Complex modal structure provides a challenging problem for the automated picking algorithm. If two modes are similar in frequency and slowness, as shown in figure 1b, the picking algorithm
Section III: Background of Study Area and Technology or Methods

For most cases in seismic processing, only simple body wave propagation is considered, and it is assumed that all frequencies represented by a waveform are traveling at the same velocity. When the velocity of the seismic wave is dependent on the frequency, the wave is considered to exhibit a dispersive phenomenon. In surface waves, dispersion is caused by the combination of a layered earth model and the half-space along which the wave propagates. No traction exists at the surface resulting in a stress free boundary. This introduces a boundary condition to the common plane wave expression:

\[
G = Ae^{-jkz} e^{i(\omega t - kr)},
\]

where, \( \gamma = \sqrt{1 - \frac{V_p^2}{\beta^2}} \),

and, \( k = \frac{\omega}{V_p} \).

From this equation it can be seen that in a broad sense, as the depth, \( z \), approaches infinity the amplitude term, \( Ae^{-jkz} \), approaches zero. The equation also shows increasing frequency, \( \omega \), included in the wave number term, \( k \), at a particular depth will cause the amplitude of the wave to decrease. In other words, lower frequency waves sample the higher velocity layers existing at depth; whereas, higher frequency waves decay and do not sample the high velocity layers existing at depth. The described phenomenon sets up a dispersive environment where lower frequency surface waves travel at a higher velocity than the high frequency surface waves. Three types of
velocities are considered, shear wave velocity of the material, $\beta$, phase velocity, $V_p$, and group velocity, $V_g$. Phase velocity refers to the motion of waves and is simply the difference in phases across a distance. Group velocity can be illustrated by adding two waves of slightly different frequencies traveling at slightly different velocities. Figure 2 shows the result of adding a wave of higher frequency and lower velocity to a wave of lower frequency and higher velocity. As can be seen in Figure 2, the result of this addition resembles a pulse. The speed at which the pulse travels is the group velocity.

![Wave Addition](image)

Fig. 2. Addition of two waves shown in top two panels with result in lower panel. Top panel displaying wave of frequency 10 Hz and velocity of 100 m/s. Middle panel displaying wave of frequency 9.9 Hz and velocity of 110 m/s. Distance shown is 500 meters.

The speed at which the wavelets travel is the phase velocity. The wavelets are modulated by a low frequency wave called a wavepacket or envelope. The speed at which the low frequency envelope travels is the group velocity. Phase velocity is usually faster than the group velocity when considering the standard model for the near subsurface. A reverse dispersion where group velocity exceeds phase velocity occurs when the upper layers of the earth are higher in velocity than the lower layers. Group and phase velocity never exceed the shear wave velocity of the material. Group velocity is expressed as the derivative, or slope of the phase velocity, where
\[ V_p = \frac{\omega}{k}, \quad \text{and} \]
\[ V_g = \frac{d\omega}{dk}. \]

A simple 2D model given in Table 1. is investigated to illustrate the dispersive nature of surface waves. Velocity specifications that are similar to the survey area shown in Figure 1 are selected. In this case, only a solution for a Rayleigh wave is considered since the majority of energy in the surface wave is usually due to this type of wave. As described above in equation 1, a Rayleigh wave exists at the half-space; in addition, it is a combination of the P wave and vertical component of the shear wave. At a minimum, a half-space and a layer beneath will set up a dispersive environment. Given the model specified in Table 1, an algorithm developed by (Herrmann, 2002) is used to determine the elastic solution for phase velocities of the Rayleigh wave. The dispersion curve with the lowest velocity, shown as a solid blue line in Figure 3, is called the fundamental mode. A solution for a second dispersion curve is included for the model and is shown as a dashed blue line in Figure 3. There can be a number of dispersion curves that exist for the model because waves can travel at different velocities for a particular frequency. Dispersion curves which exist at higher frequencies are referred to as higher order modes.

<table>
<thead>
<tr>
<th>Depth(m)</th>
<th>Vp(m/s)</th>
<th>Vs(m/s)</th>
<th>RHO(gm/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>300</td>
<td>150</td>
<td>1.80</td>
</tr>
<tr>
<td>(\infty)</td>
<td>360</td>
<td>180</td>
<td>1.82</td>
</tr>
</tbody>
</table>
Fig. 3. Dispersion curves for fundamental mode, solid blue line, and 1$^{\text{st}}$ higher mode, dashed blue line, generated from Computer Programs in Seismology (CPS) from Saint Louis University. Group velocity for the fundamental mode shown with dashed red line.

Phase velocities for all modes are bounded by the shear wave velocity of the model. Group velocity, being the derivative of the phase, can drop below the shear wave velocity of the model at the inflection point along the phase velocity curve. The point at which the minimum in group velocity occurs is known as the Airy phase.

In the shot record, the phase and group velocity look similar to Figure 4.

Fig. 4. Illustration of phase velocity ($V_p$) and group velocity ($V_g$) in a shot record.
Although phase velocity may be substantially higher, the travel time of the surface wave will be limited by the group velocity.

As stated earlier, GRB-3D is a phase-matched filtering algorithm that compresses and separates the surface wave-train from the seismic reflection. Estimating spatially varying dispersion curves is fundamental for separating the surface wave train. In its entirety, GRB-3D operates as shown in the flow depicted in Figure 5.

![Flow diagram of GRB-3D operation](image)

**Fig. 5. Overall GRB-3D operation flow**

For purposes of discussing the ERI algorithm in the context of GRB-3D, step b is explained in detail. Recall that a spatially varying velocity versus frequency volume for the near subsurface, or dispersion volume, is used for phase-matching. The dispersion volume is representative of many dispersion curves averaged from windowed areas of the survey. To generate the power
spectrum for the dispersive energy of each window of the surface wave a signal processing algorithm called beamforming is used.

The term beamforming is borrowed from radio and antenna design. Early spatial filters, such as the dish on an antenna, were designed to receive signals from one location and attenuate signals from elsewhere. The curve of the dish steers the array in such a way that the amplitude of a coherent wave front is enhanced relative to background noise. In a similar way, GRB-3D steers phase velocity (or slowness) in the frequency domain to recover the power spectrum of the dispersive energy. First, each trace, $g(t)$, is transformed to the time-frequency domain:

$$G(\omega) = \int_0^{t_{\text{max}}} g(t)e^{i\omega t} dt,$$

where $\omega = 2\pi f$.

To demonstrate this process, the shear wave model shown above in Figure 1 is used again with a different window of traces, shown in Figure 6a. Figure 6b shows the selection of synthetic traces generated from the model.

![Shear Wave Velocity at 10m depth](image1)

![Synthetic Traces in Window](image2)

**Fig. 6.** Shear wave velocity model. (a) Synthetic traces generated from shear wave velocity model. (b)
Considering there are many traces within a given spatial window, the surface wave is now represented as a function of frequency and source-receiver offset radius, $G(f, r)$, and is shown in Figure 7a. The expression can be written as:

$$|G(f, r)|e^{-ik_{\text{true}}r},$$

where, $k_{\text{true}}$, emphasizes the fact that this is the true phase of the surface wave in the data.

A cross-correlation between traces is performed in the frequency-spatial domain at each frequency for a given range of slownesses as defined by the scanning wavenumber, $k$, (ignoring amplitude):

$$G(f, s) = \int_{0}^{R_{\text{max}}} e^{-ik_{\text{true}}r} e^{ik_{s}r} dr.$$  \hspace{1cm} (9)

Given the relationship,

$$k = \frac{\omega}{V_{p}} = \omega \cdot s,$$  \hspace{1cm} (10)

equation 9 can also be written as

$$G(f, s) = \int_{0}^{R_{\text{max}}} e^{i\omega(s_{r}-s_{\text{true}})} dr.$$  \hspace{1cm} (11)

where slowness now represents the values being steered or adjusted.
Scanning across a sufficient range of slowness values images the dispersive energy of the surface wave. In Figure 7a, note that energy from the surface wave dominates the lower frequency. Correspondingly, slowness is steered across the low frequency range (3-18Hz) to image the dispersive energy, as shown in Figure 7b.

To illustrate the process of beamforming in the time domain, recall that cross correlation in the frequency domain is delay and summation in the time domain. Traces in the time domain before delay is applied, as seen in Figures 8a and 9a, are compared against traces after delay is applied (Figures 8b and 9b). At a frequency of 4 Hz, shown in Figure 8a, a scanning slowness of 0.003 s/m will align the phase (Figure 8b); and, a summation across the traces will result in a high amplitude value.
Fig. 8. Inverse FFT from temporal-frequency domain at 4Hz before delay applied. (a) Inverse FFT from temporal-frequency domain at 4Hz with slowness values that line up phase waveform (b)

When scanning at a higher frequency in the waveform, such as 10Hz in Figure 9a, the slowness values required to line up the phase increases to 0.005 s/m, as shown in Figure 9b.
Fig. 9. Inverse FFT from temporal-frequency domain at 10Hz before delay applied. (a) Inverse FFT from temporal-frequency domain at 10Hz with slowness values that line up phase waveform (b)

Recall the dispersive energy shown in Figure 7b; as slowness is steered through each frequency a beam will form where the waveform constructively interferes compared to a summation where
the waveform de-constructively interferes. The curve expressed in the dispersive energy is then characteristic of the near surface properties at that group of receivers.

ERI addresses complications associated with beamforming windowed traces. Small window size provides high spatial resolution of the surface wave velocity-frequency characteristics; however, there is a limit to using small windows. Recalling the Heisenberg-Gabor inequality, it is known that a signal cannot have a simultaneously high resolution in time and frequency. This property is a consequence of the Fourier transform and holds true for sampling in the spatial domain as well. There are a number of ways to illustrate the effect of the duality of the Fourier transform. In the extreme sense, it is known that an infinite time series transforms to a spike. Since there are not an infinite amount of receivers within a window, degradation from an ideal imaged dispersion curve always occurs. As size of the window decreases, the resolution of beamformed dispersive energy also decreases. Using the method of beamforming within a window then becomes a trade-off between using small or large windows. An additional complication is the effect of heterogeneity on the surface wave within the window. More than one fundamental mode will exist for a windowed area. Recalling that multiple modes can exist in a spatially homogeneous model, it can sometimes be difficult to determine the cause of an additional mode. It is common that multiple modes due to spatial heterogeneity will be similar in properties to one another posing a challenge for an automated picking algorithm. An algorithm such as ERI has the potential to collapse, or constrain, the energy smeared in the beamforming process.

ERI is based on the center of gravity equation. An amplitude value within a moving window of an image is reassigned to the center of gravity of that window. By taking the ratio of the sum of the distances, $r$, multiplied by the amplitudes, $A(r)$, in respect to the sum of the amplitudes, a center of gravity based on amplitude, $\hat{r}$, can be found:
\[
\hat{r} = \frac{\int r \cdot A(r) dr}{\int A(r) dr}
\] (12)

As shown in Figure 10, the amplitude value at point 1 is reassigned to the center of gravity for that window, point 2.

![Diagram of reassignment within window](image)

Fig. 10. Illustration of the ERI algorithm. Rectangle defines window; dashed circle represents an area of higher amplitude.

Once new coordinates are found for each window they are gridded on a 2D array of identical size to the 2D array of the original image. ERI can be iterated to further constrain the dispersive energy. Different window sizes and ratios can be used to customize the effect of the filter on the image. Details of the ERI algorithm are explained further in Appendix A.

**Section IV: Results of Study**

In order to illustrate the operation of GRB-3D and ERI, a derived dataset has been created from field data. A volume file representing x, y, and frequency was created in the GRB-3D analysis from field data. Next, a 3D model representing shear wave velocity in the x, y, and depth was created from a 1D elastic inversion for shear wave velocity at each receiver location. Synthetic data for a single shot was created from this model with a 3D elastic modeling algorithm.
A large window of receivers is selected from the model, shown in Figures 11a and 11b, to illustrate spatially heterogeneous effects of the surface wave and the ERI process. The beamformed image, in Figure 12a, shows two modes where the smeared image causes the automatic picking to go astray.

The beamformed image is specified by an array of 1000X60 pixels. ERI is set to use a moving window that is 20X5 pixels. The window is set to move every 10th pixel in the x direction every pixel in the y direction. Figure 12b shows the result after 2 iterations of reassignment. Automatic picking is shown as a white line in the figures.
Fig. 12. Beamformed image. (a) Reassigned beamformed image. (b)

ERI collapses the smeared energy around the modes allowing the automated picking algorithm to operate correctly. The solid white line in Figure 10b shows the reassigned pick versus the original pick, shown as a dashed white line. Lower frequency picks in the reassigned image differ slightly from the original.

GRB-3D is applied to the dataset using the dispersion curve picked before ERI; and, using the dispersion curve picked after ERI. Phase matching is shown in Figures 13a and 13b. Note the surface wave is better flattened, more coherent, and better compressed using ERI.
Fig. 13. Flattened surface wave using dispersion curve picked without ERI. (a) Flattened surface wave using dispersion curve picked with ERI. (b)

Better aligned surface waves in the phase matching stage using the reassigned curve allows for a better noise model and a more accurate subtraction, shown in Figures 14a and 14b. Artifacts from the incorrect dispersion curve, such as the ringy un-collapsed surface wave, are no longer present in the filtered mode.
Section V: Conclusions and Recommendations

An iterative approach using energy reassignment will assign points closer to the center of gravity in each mode. In this sense, ERI can be used as a mode separation technique. Separating the modes improves the picking algorithm which, in turn, improves the filtering possible in GRB-3D. To some extent, the accuracy of the method is a function of the sampling of the moving window and re-gridding. However, in our example the benefits outweigh the drawbacks. Currently, ERI is implemented into the analysis section of the GRB-3D program. ERI is compute intensive. Since the implementation; however, ERI has been optimized by a factor of ten, and it is possible
tests can be conducted using ERI in full automation for determining dispersion curves. Further optimization can be explored using a moving average and recursive operations. ERI also provides groundwork for automatic picking methods with minimal boundary parameters, although more investigation is needed towards the effect of windows on the operation. Gaussian windows could also be applied to the process. Other ways of dealing with edge effects at the image boundary could also be explored.

**Section VI: References**


Herrmann, R. B., 2002, Computer Programs in Seismology, Department of Earth and Atmospheric Sciences Saint Louis University.

Appendices:

Appendix 1.

Abstract accepted by 72nd EAGE Conference & Exhibition, Barcelona, Spain, 14-17 June 2010
Energy reassignment of an image for improved picking of dispersion curves in spatially inhomogeneous media
Craig W. Hyslop and Mamadou S. Diallo, ExxonMobil Upstream Research Company

Many algorithms have been proposed to improve the resolution of the dispersion curve through the use of different transforms to the frequency-velocity domain. The reassignment proposed in this paper is an image processing and picking technique that has the ability to overcome problems intrinsic to poor sampling and complex modal structure. An iterative approach using energy reassignment will assign points closer to the center of gravity in each mode. In this sense, energy reassignment of the image can be used as a mode separation technique. An example from within the processing flow of a surface wave mitigation algorithm is given to show the advantage in using this process on beamformed dispersive modes.
Introduction

Estimating accurate velocity dispersion curves for surveys that exhibit spatially varying surface-wave characteristics is fundamental for determining parameters needed to mitigate surface waves. Many algorithms have been proposed to improve the resolution of the dispersion curve through the use of different transforms to the frequency-velocity domain. Pedersen et al. (2003) applied the concepts of energy reassignment to multi-frequency analysis and successfully improved interpretability of the dispersion curve. Here, a direct approach to energy reassignment is taken in order to reassign energy in the 2-D image after, rather than during, the frequency-velocity transformation. In this paper, “energy” is used loosely to mean the amplitude of the image.

We use energy reassignment of the image as a means to constrain dispersion curves within the context of GRB-3D (Ground Roll Buster 3DSM), a surface-wave mitigation technique that adaptively accounts for spatial variation in surface waves (Lee et al. 2008). GRB-3D is a phase-matched filtering algorithm that compresses and separates the surface-wave train from the seismic reflection. To account for surface-wave heterogeneity, dispersion curves are calculated for windowed areas of the survey, instead of using one dispersion curve for the entire survey area. For each window, phase velocities of the dispersion curve are estimated by beamforming. Using small spatial windows ensures that heterogeneity throughout the spatial domain of the survey area is captured. However, the duality of the Fourier transform dictates that as the number of samples within the spatial window decreases, the width of the beamformed dispersion curve in the frequency-velocity domain increases. In addition to the resolution issue, the character of the surface wave within the windowed area may exhibit more than one dispersive mode, or there could be spatial heterogeneity within the windowed area. A loss of resolution in the frequency-velocity domain combined with a complex modal structure makes picking frequency-velocity pairs for separate modes difficult.

Previous reassignment algorithms follow the method proposed by Kodera et al.(1976) by reassigning the time-frequency location of each point in the spectrum to a new location based on the partial derivative in time of the complex short-time Fourier Transform. Pedersen et al. (2003) adapted their method to Multi-Frequency Analysis (MFA). Using reassigned MFA, each trace within their array was transformed, and the resulting frequency-velocity planes were stacked to create the complete reassigned 2D semblance of the frequency-velocity domain for the area. It is important to note that Pedersen et al. and others before them were correcting systematic errors inherent to the short-time-frequency transform. The reassignment proposed in this paper is an image-processing and picking technique that has been decoupled from the transform processes to overcome problems intrinsic to poor sampling and complex modal structure. By constraining dispersive modes within the image, interpretability is improved and picking separate modes becomes less difficult. In this presentation, we evaluate the accuracy of energy reassignment and give an example from the processing flow of GRB-3D to show the advantage in using this process on beamformed dispersive modes. The example exhibits two smeared modes in the surface wave.

Method

We follow basic applications introduced by Kodera et al. (1976) to directly reassign energy (Figure 1). Essentially, a point in the image space is moved to the center of gravity within a window. In relation to the beamformed image $G_B$, the energy at point $(s_p', f')$ that is reassigned to the center of gravity $(\hat{s}_p, \hat{f})$ can be described as:

$$\hat{s}_p(s_p', f') = s_p' - \int \int \phi(s_p, f) \cdot s_p \cdot G_B(s_p' - s_p, f' - f) ds_p df$$

(1)
\[
\hat{f}(s', f') = f' - \frac{\int \phi(s_p, f) \cdot f \cdot G_B(s' - s_p, f - f) ds_p df}{\int \phi(s_p, f) \cdot G_B(s' - s_p, f - f) ds_p df}
\] (2)

where the moving window is defined by \( \phi(s_p, f) \). The reassigned beamformed image \( R_B(s_p, f) \) is the total contribution of all the center of gravity values calculated within the windows:

\[
R_B(s_p, f) = \int G_B(s' - s_p, f') \delta(s_p - \hat{s}_p(s_p, f')) \delta(f - \hat{f}(s_p, f')) ds_p df'
\] (3)

**Figure 1:** Principle of reassignment method. The red line indicates the crest of a beamformed image, while the blue line represents the extent of the mode. The dashed circle indicates the moving window. For each window, the energy of \( (s_p, f) \) is moved to the center of gravity \( (\hat{s}_p, \hat{f}) \).

To implement the reassignment as an algorithm, the beamformed image is represented by a 2D array of energy values in which the axes correspond to frequency and slowness. Coordinates for the calculated center of gravity in each sampled window are fit to a 2D array with dimensions identical to the original image using Delaunay interpolation. The following pseudo-code describes the algorithm used:

1: loop over n windows in 2D array
2: calculate \((f_{\text{center}}, s_{\text{center}}, e_{\text{center}})\)
3: end loop
4: grid \((f_{\text{center}}, s_{\text{center}}, e_{\text{center}})\)

The proportions of the image are maintained by using a boxcar window that is circular and symmetrically expressed in both the frequency and slowness axes. Increasing the size of the window allows for a point to be moved a greater distance within that window towards the center of gravity; however, similar to what was noted by Pedersen et al (2003), modes that are close together will be smeared into one mode. Decreasing the size of the window to an area smaller than the energy crests between modes prevents smearing by reassignment. The benefits of such a technique are minimal as energy is reassigned only a short distance within the moving window. A solution to this limitation is to take an iterative approach using appropriately small moving windows. Each successive reassignment assigns energy closer to the center of gravity in each mode. In this sense, energy reassignment of the image can be used as a mode-separation technique. The accuracy of the reassignment is dependent on the sampling increment of the moving window across the 2D array. Using an increment in which the center of the moving window samples a subset of the total number of pixels in the image increases the speed of the algorithm, but also results in slightly less accurate reassignments.
Examples

To demonstrate the accuracy of this reassignment method a comparison is made between the positions of maximum energy in a Gaussian distributed beam and its reassignment. A Gaussian beam is constructed through a 2D space and is represented by a 50X500 array. Coordinates for the center of gravity are sampled every pixel in the frequency direction and every tenth pixel in the slowness direction (Figure 2). The correlation coefficient between the crests of the original and reassigned images is 1.0 in the frequency dimension and 0.9998 in the slowness dimension. Disparity in the slowness dimension results from the fact that the maximum energy in the image may only exist in one pixel. Therefore, some information is lost when sampling every tenth pixel.

![Figure 2: Top Left) Gaussian beam represented by a 50X50 matrix. Top Right) Coordinates of reassigned Gaussian beam after first iteration. Bottom Left) Reassigned Gaussian Beam after reassignment. Bottom Right) Maximum energy picked for both beams: blue – original Gaussian beam; and, red – reassigned Gaussian beam.](image)

We use an example of a beamformed image produced by GRB-3D\textsuperscript{SM} exhibiting two nearby modes in the surface wave to evaluate energy reassignment of the image (Figure 3). Picking is automated by an algorithm that steps through each frequency in the image and searches for the maximum energy in the slowness axis within a range defined by half the mode width. In this particular example, the structure of the modes is such that the picking algorithm has skipped from the primary mode across to a higher velocity mode. Reassignment is performed in the same manner as in the previous example using two iterations of a moving window that is smaller in size than the separation between the two modes. The reassignment sufficiently constrains each mode that the automated picking remains on the primary mode (Figure 3).
**Figure 3:** Left) Automated picking performed on original beamformed image. Right) Automated picking performed on reassigned beamformed image.

**Conclusions**

Energy reassignment allows the dispersive modes to be constrained, thereby improving frequency-velocity pair picking. By implementing an iterative approach, reassignment effectively separates modes. This improves the ability of surface-wave mitigation programs, such as GRB-3D$^\text{SM}$, to match phases of separate modes properly. Directly reassigning the frequency-slowness image is computationally efficient, thus facilitating interactive analysis and processing of surface waves. Certain prudence needs to be taken in selecting the sampling interval for the moving window to ensure that the reassignment remains accurate. While this reassignment process is presented as a method to improve mode picking within GRB-3D$^\text{SM}$, there may be other applications for this type of reassignment, including velocity analysis and high resolution Radon analysis.

**Acknowledgements**

Thanks to Warren Ross, Sunwoong Lee, Bill Curry, and Chris Khron for many useful discussions.

**References**


Lee, S., and W. S. Ross, 2008. 3-D mitigation of surface-wave noise in spatially inhomogeneous media, 78th annual SEG meeting, Expanded Abstracts, 2561-2565

Appendix 2.

Note: The dynamic method for updating a velocity dependent time window was completed; however, more substantive research could be applied towards the task mentioned as the first possible addendum, energy reassignment.
This is a proposal to develop a dynamic method for updating a velocity dependent time window for polarization filtering used in mitigating surface wave noise in spatially inhomogeneous media. Development will be done at the ExxonMobil Upstream Research Company within the Signal Enhancement Section of the Geophysics Division. The work will apply techniques used for 3-D mitigation of surface-wave noise in spatially inhomogeneous media developed by Sunwoong Lee and Warren Ross (Lee et al., 2008) to polarization filtering techniques for surface wave mitigation developed primarily by Mamadou S. Diallo and Warren S. Ross (Diallo et al., 2008).

Possible addendums to the above proposal include: 1.) Using methods of energy reassignment to improve the group velocity measurements of surface wave dispersion curves in the time-frequency domain; and 2.) planting weighted seeds of manually picked dispersion curves in a survey area to guide the remaining automated picking.

Mamadou S. Diallo has developed a method of constraining polarization filtering to the part of the data most dominated by surface waves (Diallo et al., 2008). An initial step includes windowing the surface wave-train of a shot-gather. Currently, a single window is used to define the extents of polarization filtering for all shot-gathers within the survey area. To ensure the inclusion of the surface wave-train for all shot-gathers, the lowest frequency phase velocity from the survey area containing the fastest velocities is used. This defines the broadest moveout needed in the survey area for windowing the surface wave-train. However, using a static window for polarization filtering in areas of slower surface wave velocities will include reflected energy outside the surface wave-train and possibly harm signal. A dynamic window that adapts to velocity variations of the surface wave-train throughout the survey area will minimize the amount of reflected energy affected by the polarization filter.

Applying energy reassignment in the time-frequency domain to dispersion curves will improve precision of the automated group velocity measurements. Using planted seeds of manually picked dispersion curves throughout the survey area will ensure against the automated routine picking velocities from a dispersion curve associated with a secondary mode.

Several types of data will be required to complete the proposed Capstone Project. Seismic data will be available from the ExxonMobil Friendswood test site. A binary “volume” file describing the spatial velocities in the survey area will be produced by a 3-D surface-wave mitigation program (GRB3DSM). Code and other output from GRB3DSM and the polarization filtering routine will also be available for development of new code.

Initial research for the project is currently in progress and new addendums can be implemented as needed. Accessing seismic data will be the first step in the workflow. The application of polarization filtering to the Friendswood dataset will be ongoing as new code is developed. New use of algorithms will be noted throughout the project and analysis of the results will be reported in written and presentation format (Fig. 1.)
An increased preservation of signal while maintaining noise mitigation effectiveness when compared to the previous static windowing method is expected from the proposed development of dynamic windowing. The improved resolution in the data from good noise mitigation will continue help the interpreters produce better informed decisions about the horizons of interest.
References

Sunwoong Lee and Warren S. Ross, 2008, 3-D mitigation of surface-wave noise in spatially inhomogeneous media: SEG Las Vegas 2008 Annual Meeting, 2561-