Asymptotic calculations of the Biot’s waves in porous layered fluid-saturated media
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Summary

Biot’s poroelasticity model predicts the existence of a slow compressional (diffusion) wave due to the relative flow of pore fluid with respect to the rock solid. This phenomenon opens an opportunity for investigation of fluid properties of the hydrocarbon-saturated reservoirs, in particular fluid mobility, from seismic amplitude. Low-frequency asymptotic description of the Biot’s model provides relatively simple form of reflection and transmission coefficients. In case of normal incident the coefficients only depend on impedance contrast and small dimensionless parameter that is a product of fluid density, viscosity and rock permeability. All parameters are measurable. The goal of the present study is to use asymptotic solution and propagator matrix method for investigation of the slow P wave effect on reflectivity of the porous layered fluid-saturated media.

Introduction

Dependence of seismic reflections on frequency from a gas-water boundary in a porous sand reservoir was calculated by Dutta & Ode (1983) using exact Biot’s model (1956). It was also shown by Dutta & Ode (1979a, b) that relative fluid movement becomes negligible at seismic frequencies if the porous material is homogeneous, while a heterogeneous fluid saturation leads to a substantial effect on attenuation. Carcione et al. (2003) also demonstrated a significant seismic attenuation due to heterogeneities in either permeability or fluid saturation. Thus, it is well known that strong attenuation of the transmitted P-waves is usually associated with heterogeneities in permeability and fluid saturation. However, besides the attenuation of the transmitted P-waves the effect of both fluid saturation and fluid mobility on seismic reservoir reflectivity has not been extensively studied.

Silin and Goloshubin (2008, 2009) carried out a low frequency asymptotic analysis of Biot’s poroelasticity. They used both fluid flow and scattering mechanisms to derive a frequency dependent reflection. In this case, the reflection and transmission coefficient are expressed as power series of the square root of a dimensionless parameter:

\[ \varepsilon = i \frac{\rho_f \kappa \omega}{\eta}, \]

where, \( \rho_f \) is fluid density, \( \kappa \) is permeability, \( \eta \) is fluid viscosity, \( \omega \) is angular frequency, and \( i \) is the imaginary unit. A brief description of asymptotic calculation is summarized in the next section.

Asymptotic calculation

In case of Fast P incident wave, the reflection and transmission coefficients from Fast P wave to Fast P wave are denoted as \( R_{FF} \) and \( T_{FF} \); the reflection and transmission coefficients from Fast P wave to Slow P wave are denoted as \( R_{FS} \) and \( T_{FS} \). They have the following asymptotic forms for normal incident P wave:

\[ R_{FF} = R_0 + R_{FF}^{1-i} \frac{1 + i}{\sqrt{2}} \sqrt{|\psi|}; \]
\[ T_{FF} = 1 + R_0 + T_{FF}^{1-i} \frac{1 + i}{\sqrt{2}} \sqrt{|\psi|}; \]
\[ R_{FS} = R_{FS}^{1-i} \frac{1 + i}{\sqrt{2}} \sqrt{|\psi|}; \]
\[ T_{FS} = T_{FS}^{1-i} \frac{1 + i}{\sqrt{2}} \sqrt{|\psi|}; \]

where \( R_0 \) is the zero order classical reflection coefficient, and the first order reflection and transmission coefficients \( R_1^{FF} \) and \( T_1^{FF} \) have the form:

\[ R_1^{FF} = Z_2 (T_1^{FS} - R_1^{FS}) \frac{Z_1 + Z_2}{Z_1 + Z_2}; \]
\[ T_1^{FF} = Z_1 (R_1^{FS} - T_1^{FS}) \frac{Z_1 + Z_2}{Z_1 + Z_2}. \]

Here \( Z_1 \) and \( Z_2 \) are the acoustic impedance of medium 1 and medium 2 respectively.

The first order reflection and transmission coefficients \( R_1^{FS} \) and \( T_1^{FS} \) have the form:

\[ R_1^{FS} = A \left( \gamma_{M1} \gamma_{M2} \right)^{1/2} + \gamma_{M2}; \]
\[ T_1^{FS} = A \left( \gamma_{M1} \gamma_{M2} \right)^{1/2} + \gamma_{M1}, \]

here, subscript 1 and 2 indicates medium number. Other internal descriptions related to fluid and solid are:

\[ \gamma_{\beta} = M \left( \beta_f \phi + \frac{1 - \phi}{K_{RG}} \right); \]
\[ \gamma_{M} = 1 - \frac{(1 - \phi) K}{K_{RG}}; \]
\[ A = \left[ \frac{\gamma_{M1}}{\gamma_{M1}} + \gamma_{M2} \right] \frac{2Z_1Z_2}{Z_1 + Z_2}; \]
\[ D = \frac{Z_1Z_2}{\gamma_{M1} \gamma_{M2}} \left[ \frac{1}{\gamma_{M1} \gamma_{M2}} \frac{1}{\gamma_{M1}} \frac{1}{\gamma_{M2}} \right] \frac{v_{s1} v_{s2}}{M_1 \sqrt{\gamma_{M1}} + \gamma_{M1}} + \frac{1}{\gamma_{M1}} \frac{v_{s2}}{M_2 \sqrt{\gamma_{M2}} + \gamma_{M2}}. \]
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where,

\[ \beta_j = \frac{1}{K_f}; \quad \gamma_f = \frac{\varepsilon_f}{\varepsilon_l}; \quad \gamma_p = \frac{\rho_p}{\rho_f}; \quad M = K + \frac{4}{3} \mu. \]

\[ K_{ge} = \frac{K_s}{1 - \phi}; \quad \text{and} \quad K_{ge} = \frac{K_g}{1 - \phi}. \]

Here \( K_s \) is the bulk modulus of fluid, \( K_g \) is the bulk modulus of solid grain, \( K \) is the dry rock bulk modulus, \( \phi \) is porosity, \( \rho_f \) is the fluid density, and \( \rho_p \) is the bulk density.

A summary of all necessary input properties is demonstrated in Table 1. It can be seen that the input parameters are grain bulk modulus \( K_g \), dry rock bulk modulus \( K_dry \), dry rock shear modulus \( \mu_{dry} \), bulk modulus \( K_o \), grain density \( \rho_g \) and fluid density \( \rho_f \). These parameters are routinely used in fluid substitution technique based on Gassmann’s equation. The asymptotic description of the Biot’s model includes two additional parameters (rock permeability \( \kappa \) and fluid viscosity \( \eta_f \)). Thus, it allows besides realization of the fluid substitution technique to provide an investigation of the influence of the permeability (fluid mobility) to seismic response. All input parameters can be acquired from log data and laboratory measurements. Hence, it makes the asymptotic description of the Biot’s model more practical for application.

<table>
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<tr>
<th>( K )</th>
<th>( \rho_g )</th>
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<th>( \mu_{dry} )</th>
<th>( \phi )</th>
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Table 1. Input parameters and units for asymptotic solution.

Furthermore, velocity of Fast P wave and Slow P wave, \( V^F \) and \( V^S \); attenuation coefficients of Fast P wave and Slow P wave, \( a^F \) and \( a^S \) (in unit of m\(^{-1}\)) can be calculated from:

\[ V^F = v_f \sqrt{1 + \frac{\varepsilon_f}{\gamma_f}}; \]

\[ V^S = v_f \sqrt{\frac{2\varepsilon_f}{\gamma_f + (\gamma_f)^2}}; \]

\[ a^F = \frac{\omega}{v_f} \sqrt{\frac{\gamma_f}{\gamma_f + (\gamma_f)^2}} \frac{s^F}{\beta f}; \]

\[ a^S = \frac{\omega}{v_f} \sqrt{\frac{\gamma_f}{2\varepsilon_f}}; \]

where,

\[ v_f = \frac{M}{\rho_f}; \quad v_f = \frac{M}{\rho_f}; \quad s^F = \left( \frac{\gamma_f}{\gamma_f} \right) \frac{\gamma_f}{\gamma_f}. \]

Asymptotic solution also provides the reflection and transmission coefficients in case of Slow P wave as incident wave, where the reflection and transmission coefficients for converted Slow to Slow P wave are denoted as \( R^SS \) and \( T^SS \); and the reflection and transmission for converted Slow to Fast P wave are denoted as \( R^{SF} \) and \( T^{SF} \).

They have the asymptotic forms below:

\[ R^SS = \frac{-\chi_0^S}{\chi_0^S} \frac{1}{\chi_0^S} \left( M^S k^S e^S + \chi_0^S M^S k^S e^S \right) \frac{1}{\chi_0^S} \frac{1}{\chi_0^S} \left( M^S k^S e^S + \chi_0^S M^S k^S e^S \right) \]

\[ T^SS = \frac{Z_2 (-1 - R^SS + T^SS)}{Z_1 + Z_2}, \]

\[ R^{SF} = \frac{Z_2 (-1 - R^SS + T^SS)}{Z_1 + Z_2}, \]

\[ T^{SF} = \frac{Z_2 (-1 - R^SS + T^SS)}{Z_1 + Z_2}, \]

where,

\[ k_0 = \frac{1}{\chi_0^S} \sqrt{\gamma_f + (\gamma_f)^2}; \quad \chi_0^S = \gamma_f + \frac{\gamma_f}{\gamma_f} \frac{1}{\gamma_f}. \]

Propagator matrix method

Following the description of Robinson (1967), we demonstrate a normal incident Fast P wave and Slow P wave propagation through porous layered media using the Thomson (1950) and Haskell (1953) propagator matrix method.

According to the boundary condition in Figure 1, we can obtain the relationships between all waveforms at interface \( j \) with the corresponding reflection and transmission coefficients. We use \( rff \) to represent the reflection coefficient of Fast P wave to Fast P wave while the incident wave is downgoing, and \( rffup \) to represent the reflection coefficient of Fast P wave to Fast P wave while the incident wave is upgoing. Thus, \( rff = u_f(t+1)/u_f(t-1) \) and \( rffup = d_f(t+1)/u_f(t-1) \). Similar denotations are used for other reflection and transmission coefficients.

The time delay for Fast P wave travel through any layer \( j \) is taken to be \( t \) unit of time. The time delay for Slow P wave travel through any layer \( j \) is taken to be \( t \) unit of time.
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Thus, \( \tau \) depends on the relative velocity of Fast P wave and Slow P wave, i.e.,

\[
\tau_j = \frac{v_{fast,j}}{v_{slow,j}}.
\]

Also, define \( D_j(z) \) as the z-transform of \( d_l(t) \), i.e.,

\[
D_j(z) = \sum_{t=0}^{n} d_j(t) \cdot z^{-t}.
\]

\( n \) is the total number of samples in the time series of \( d_j(t) \). And similarly define \( D_j'(z) \), \( U_j(z) \), \( U_j'(z) \) as the z-transform of \( d'_j(t) \), \( u_j(t) \), \( u'_j(t) \), respectively.

Then, we can obtain for any interface \( j \):

\[
\begin{bmatrix}
D_{j+1}(z) \\
D'_j(z) \\
U_{j+1}(z) \\
U'_j(z)
\end{bmatrix}
= z^{-j} \begin{bmatrix}
D_j(z) \\
D'_j(z) \\
U_j(z) \\
U'_j(z)
\end{bmatrix}
\begin{bmatrix}
M_j
\end{bmatrix},
\]

where, the four by four matrix \( [M_j] \) is the propagator matrix that communicates the waveforms between layer \( j \) and \( j+1 \). Each matrix element of \( [M_j] \) is also in z-transform, i.e., polynomials of \( z \). Thus, for wave propagation in multi-layered media (Figure 2), we can obtain:

\[
\begin{bmatrix}
D_{j+1}(z) \\
D'_j(z) \\
U_{j+1}(z) \\
U'_j(z)
\end{bmatrix}
= \prod_{j=0}^{k} \begin{bmatrix}
D_j(z) \\
D'_j(z) \\
U_j(z) \\
U'_j(z)
\end{bmatrix}
\begin{bmatrix}
M_j
\end{bmatrix},
\]

and the multiplication between any two matrix elements in \( [M_j] \) is a convolution of their polynomial coefficients.

By setting \( D_0(z) = 1, D'_0(z) = 0, U_{k+1}(z) = 0, \) and \( U_{k+1}'(z) = 0 \), we can obtain \( U_0(z) \) as the reflectivity series of an impulse Fast P wave traveling through the multi-layered media, with mod conversion to Slow P wave and multiples taken into account.

**Example 1: Homogeneous vs. inhomogeneous fluid saturation**

In this example, we calculate the response from two types of fluid saturated multi-layered media. Both media are porous permeable sandstone fully saturated by some fluid. However, one is only saturated by water, which corresponds to a homogeneous fluid saturation, the other one is alternatively saturated by gas and water (Table 2), which leads to inhomogeneous fluid saturation. The reflectivity series from these two types of media are plotted as a function of frequency (Figure 3 and Figure 4).

It can be seen that for homogeneous fluid saturated media, there is almost no visible Slow P wave effect, while for inhomogeneous fluid saturated media, a significant Slow P wave effect exists. And this Slow P wave effect varies for different frequencies. A longer and stronger Slow P wave effect exists for lower frequency, and shorter and weaker Slow P wave effect exists for higher frequency. Thus, it can be expected in seismic profile some energy will appear below an inhomogeneous fluid saturated reservoir for low frequencies, but disappear for higher frequencies. Such phenomenon is similar to the low frequency shadows observed by instantaneous spectral analysis technique, demonstrated by Castagna et al. (2003). Hence, we think
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that Slow P wave may be a major cause for the low frequency shadows.

Furthermore, it is well known that low frequency shadows are always associated with gas reservoir. We think the reason for this association is because that free gas in the gas reservoir will induce some degree of inhomogeneity by the form of gas bubbles, and consequently enhances the Slow P wave effect. This effect appears in a seismic profile like shadows beneath the reservoir zone at lower frequencies. At higher frequencies, since Slow P wave attenuates quickly, those shadows would disappear. We must confess that it is not adequate to draw a conclusion at this stage, and more evidence towards this postulate will need to be carried out.

Example 2: Permeable vs. impermeable media

Both models in Figure 3 are porous permeable media, the permeability were taken to be 2 darcy (Table 2). If the media becomes low permeable, Slow P wave effect is significantly weakened (Figure 4).

Conclusions

Dynamic modeling on multi-layered media was applied based on asymptotic description of Biot’ model and propagator matrix technique. The reflectivity series as a function of frequency for different types of fluid saturation and permeability are obtained. A strong Slow P wave effect is observed for low frequency, high permeability, and inhomogeneous fluid saturated media. Furthermore, due to the similarity between Slow P wave phenomenon and low frequency shadows observed by instantaneous spectral analysis technique, we think that Slow P wave may be the major cause for these shadows.

Acknowledgement

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<th>$\phi$ [darcy]</th>
<th>$\kappa$ [Gpa]</th>
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Table 2: Input parameters for the porous permeable inhomogeneous gas, water alternatively saturated media. One-way travel time for Fast P wave in each layer is 0.1 ms. Reflectivity results for 7 layers are plotted in Figure 3 (b).
EDITED REFERENCES
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REFERENCES